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Signaling in Monetary Policy near the Zero Lower Bound

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Abstract

What are the consequences of asymmetry of information about the future state of the economy between a benevolent Central Bank (CB) and private agents near the zero lower bound? How is the conduct of monetary policy modified under such a scenario? We propose a game theoretical signaling model, where the CB has better information than private agents about a future shock hitting the economy. The policy rate itself is the signal that conveys information to private agents in addition to its traditional role in the monetary transmission mechanism. We find that only multiple "pooling equilibria" arise in this environment, where a CB privately forecasting a contraction will most likely follow a less expansionary policy compared to a complete information context, in order to avoid making matters worse by revealing bad times ahead. On the other hand, a CB privately forecasting no contraction is most likely to distort its complete information policy rate, the consequences of which are welfare detrimental. However, this is necessary because deviating from the pooling policy rate would be perceived by private agents as an attempt to mislead them into believing that a contraction is not expected, which would be even more harmful for society.

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1 Introduction

Central Banks (CBs) around the world confront the occurrence of the zero lower bound (ZLB) and its effect on monetary policy. In many countries, CBs have actually lowered their policy rates to the ZLB in an attempt to conduct as expansionary a policy as possible, but once the ZLB is reached, the policy loses its ability to stimulate the economy, at least with traditional instruments.

Arguably, a CB would seek to prevent its policy rate from reaching the ZLB if there were some means by which it could convey positive or optimistic information to private agents regarding the future state of the economy. Many real situations fall within the following scenario: Fearing that a recession is imminent, private agents attempt to cut expenditure unaware that their collective action will reduce current demand and production, leading to unemployment and a current contraction of the economy. If the CB has better information about future events, or if private agents believe it does, current monetary policy will signal the future status of the economy. We develop a game theoretical model to explore the macroeconomic implications of situations where the ZLB is likely to be reached.

In the signaling model developed, the CB perfectly foresees whether a contractive shock will hit the economy in the future. Even when it would like to reveal its private information, arguably when it foresees no contraction, the CB needs to consider the informational disadvantage that private agents face and how its actions would be interpreted. In particular, the role of the policy rate in signaling the CB's private information becomes important, beyond its traditional role of affecting real rates under price rigidities. And when privately foreseeing a future contraction, would the CB attempt to convey optimistic beliefs?

In a complete information situation where both the CB and private agents perfectly foresee whether a contractive shock will affect the economy and under price rigidities, the CB's policy is straightforward. If a contraction is not expected there is no need to implement an expansionary policy, whereas if a contraction is expected, the CB reduces the policy rate in order to reduce real rates. The rationale is the following: When private agents believe a recession will hit the economy, they optimally attempt to save for the future. The attempt to save is futile, however, in a homogeneous agent model, as the credit market needs to clear. With flexibility of prices, the attempt to save does not cause any problems because it will reduce the real interest rate enough such that agents are discouraged from doing so. Things change however with price rigidities because current prices may not fall sufficiently to discourage agents from saving. Their attempt to save will generate a collective drop in expenditure that leads to insufficient demand and idle resources. This situation could be avoided by reducing the policy rate sufficiently, but when the ZLB is reached, optimal allocations are unachievable.

Introducing asymmetric information into the environment begs the question of how a benevolent CB would conduct monetary policy. Specifically, when the CB privately foresees a recession, does it have an incentive to induce private agents not to cut down on expenditure aggressively? If by *not reducing* the policy rate to the ZLB the CB conceals its information, leaving private agents with uncertainty about the future, they might entertain the possibility that the economy will not enter a recessive phase and this would prevent the sort of "coordination failure" described above. In effect, the CB would succeed in preventing the full manifestation of an externality, as agents fail to internalize the collective consequences of their private decisions to save. For such a situation to be optimal, the CB type not foreseeing a contraction must choose the same policy rate as when it foresees a future contraction, to preserve uncertainty for private agents. This "pooling equilibrium" is the only type of equilibrium sustained in the model's environment.

Under a pooling equilibrium, in general, the CB type forecasting a contraction would induce a higher welfare compared to the ZLB situation, and we show that there is a large set of such equilibria not refinable by traditional arguments, with policy rates ranging from low rates near the ZLB to high rates, which would be contractive in the absence of asymmetric information. The reason why the resulting outcomes are not contractive even with high real rates is that the expectation channel is strong for private agents, outweighing the contractive effects. Our results then suggest that a CB privately foreseeing a recession will follow a less expansionary monetary policy compared to a complete information context in order to avoid making matters worse by further decreasing private expenditure and deepening the contraction. In these equilibria, the CB type that foresees no contraction naturally chooses the same pooling rate. Because off-equilibrium beliefs are unrestricted, they are assumed to be initially rather pessimistic in nature. What we attempt to capture with this assumption is the fear that private agents have when they are uncertain about the CB's intentions. If they observe a policy rate different than the one prescribed by the equilibrium, they will believe that a contractive shock is imminent and would attempt to save. The CB, seeking to avoid such a situation, will not deviate from equilibrium. We also discuss other plausible less pessimistic off-equilibrium beliefs, and arrive to similar conclusions.

The crucial assumption that the CB has private information about the future state of the economy, or that private agents believe it does is supported by the literature. Romer and Romer [2000] found that the Federal Reserve indeed possesses better information than private agents about future values of both inflation and output.

Peek et al. [2003] find also that the Federal Reserve has an informational advantage over the public and that this is due to its confidential supervisory knowledge about non-publicly traded institutions. Pedersen [2015] finds for the case of Chile that the short-term inflation expectations of private forecasters are influenced directly by the CB's forecasts.[1] Hubert [2015] finds that inflation forecasts in *real time* by CBs in Sweden, the United Kingdom, Canada, Switzerland, and Japan influence private inflation forecasts, thus supporting the possession of private information by CBs.

Many studies link the possible asymmetry of information between the CB and private agents to the "price puzzle," identified first by Sims [1992] and Eichenbaum [1992]. Using time series methods, these studies found that a cut in the policy rate induces a negative effect on inflation in the short run, contrary to the expected effects under the traditional transmission mechanism of monetary policy. Using the term "signaling channel," several recent papers have attempted to explain this puzzle, among other related questions. Melosi [2015] and Tang [2015] studied environments where uninformed agents take into account surprise changes in the policy rate to update their information about the fundamentals of the economy, which is assumed to be private information held only by the CB. Baeriswyl and Cornand [2010] and Walsh [2007] have also studied this signaling channel. Yet, these papers have not examined the strategic interactions that may arise between the CB and private agents under this informational asymmetry. Other papers investigating informational issues for CBs and the dual role of the interest rate is such settings include Vickers [1986], Aoki [2003], Gust et al. [2015], and Frankel and Kartik [2017]. To the best of our knowledge, the present study is the first to develop a *game theoretical* signaling model between the CB and private agents where the CB has private information about the future state of the economy. The CB as benevolent policy maker maximizes aggregate social welfare; it is the externality present among private agents as they fail to internalize the aggregate effects of their individual decisions regarding savings that allows the study of the agents as separate players with different objective functions. This situates the model among traditional signaling game theory models studied extensively in other settings.

The model's environment belongs to the New Keynesian tradition, which is now common and too vast to cite here. In particular, this study is related to Benigno [2009] and Mankiw and Weinzierl [2011], who develop similar models to explore different issues in New Keynesian settings. However, neither of these papers develop a signaling model to understand the allocative and welfare implications of the asymmetry of information and the strategic interactions among private agents and the CB.

[1] In particular, private forecasters incorporate the CB's forecast for output growth only when it is published in the second half of the year, when the CB may have an informational advantage due, for example, to data revisions.

The rest of this document is organized as follows: Section 2 presents the macroeconomy as well as the strategic elements that define the game. Section 3 develops the complete information benchmark where both the CB and private agents foresee which type of shock will hit the economy, as well as the asymmetric information benchmark leading to the existence of pooling equilibria. This section also presents the robustness analysis and another benchmark case where the CB and the public are symmetrically informed about the possibility of a future contraction. Finally, Section 4 concludes.

2 The Model

2.1 Environment

There is a continuum of households (HH) indexed by $j \in [0,1]$ and live for two periods $t = 0, 1$. Each produces a given variety $\ell \in [0,1]$, and each consumes all varieties that are aggregated in a Dixit-Stiglitz type index c_t^j. HH aim to maximize:

$$u^j = u\left(c_0^j\right) + \beta \mathbb{E} u\left(c_1^j\right) \tag{2.1a}$$

where $\mathbb{E}$ refers to an expectation over an aggregate productivity shock explained later. The following properties are satisfied for the period-utility function in (2.1a):

$$u'(c_t^j) > 0, u''(c_t^j) < 0, \lim_{c_t^j \to 0} u'(c_t^j) = +\infty \tag{2.1b}$$

In each period, HH are endowed with a technology allowing the transformation of n_t^j units of time into y_t^ℓ units of variety ℓ:

$$y_t^\ell = \theta_t n_t^j, \tag{2.2}$$

where θ_t is productivity and the time endowment for HH is normalized to unity. At the beginning of period 0, HH observe $\theta_0 = 1$ and expect also $\theta_1 = 1$. They set prices for the variety they produce in period 0 facing a given policy rate $i_0 > 0$ over bonds to be defined later.[2]

After setting prices for period 0, HH learn that the economy may suffer a contraction in productivity in period 1 with the following prior distribution:

$$\theta_1 = \begin{cases} 1, & \text{with probability} \quad q \\ 1-\Delta, & \text{with probability} \quad 1-q, \end{cases} \tag{2.3}$$

with $q \in [0,1]$. We assume that HH are unable to modify prices in period 0 upon arrival of new information over θ_1. We make the assumption that the CB perfectly forecasts privately the value of θ_1 and sets the policy rate i, potentially different from the initial i_0.

Using the Dixit-Stiglitz index for varieties of consumption of household j, it is possible to show that the price level P_t in each period is a particular aggregate of prices set by all HH and that demand of variety ℓ, $c_t^j(\ell)$, is isoelastic:

$$P_t = \left[\int_0^1 P_t(\ell)^{1-\eta} d\ell\right]^{\frac{1}{1-\eta}} \qquad c_t^j(\ell) = \left(\frac{P_t(\ell)}{P_t}\right)^{-\eta} c_t^j, \tag{2.4a}$$

where $\eta > 1$ is the constant elasticity of substitution among varieties. Because HH set prices at the beginning of period 0 and are unable to modify them later on during that period, the price set by each HH will be the same and equal to the price level, something deduced from (2.4a). We denote this price level as $\bar{P} = P_0 = P_0(\ell)$. Once HH observe i they may update their beliefs from (2.3) and then maximize (2.1a) choosing $c_t^j, c_t^j(\ell), P_1(\ell), n_t^j$ and bonds B^j, subject to:

$$\bar{P}c_0^j + B^j = \bar{P}y_0^\ell, \quad y_0^\ell = n_0^j, \quad 0 \leq n_0^j \leq 1 \tag{2.5a}$$

$$P_1 c_1^j = (1+i)B^j + P_1(\ell)y_1^\ell, \quad y_1^\ell = \theta_1 n_1^j, \quad 0 \leq n_1^j \leq 1, \tag{2.5b}$$

taking as given P_t and i. All real variables are denoted by lower-case letters and nominal variables by upper-case letters; the only exception is the nominal interest rate.[3]

[2]The initial policy rate i_0 is exogenous and is assumed to be strictly positive in order for the CB not be constrained later in terms of conducting expansionary monetary policy when information about future events is revealed.

[3]To simplify the analysis, we have not modeled explicitly the demand of money. Here a cash-less version of a monetary economy is assumed as in Michael Woodford [2003]. It can be modeled in an ad-hoc form by assuming that it is used for transaction purposes, this is presented in Appendix A. Money supply in period 1 will serve as a nominal anchor for P_1 as prices are fully flexible.

CB maximizes aggregate welfare, from (2.1a):

$$\int_0^1 u^j dj = \int_0^1 u(c_0^j)dj + \beta \int_0^1 u(c_1^j)dj \tag{2.6}$$

by choosing $i \in \Re^+$. Note that CB may be constrained by the ZLB.

2.1.1 Timing of the Game

The timing of the game is portrayed in Figure 1. Initially in period 0 the interest rate i_0 is set; at that moment HH do not yet expect any contraction in period 1 and they set prices $P_0(\ell)$. Then, news of a possible contraction in period 1 becomes available. While CB perfectly forecasts θ_1, HH have only the prior distribution in (2.3). CB may change the interest rate to i and then HH decide how much to consume and how much to save for period 1. In period 1, the actual value of θ_1 is realized and revealed to all and HH choose prices and consumption.

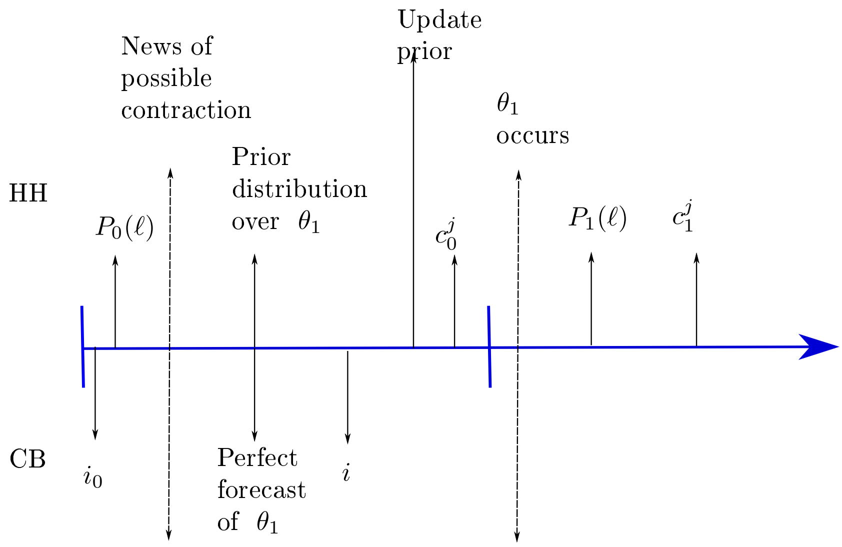


Figure 1: Timing. There are two periods. Given $i_0 > 0$, HH set prices for period 0, then news of a possible contraction in period 1 emerge. While CB perfectly forecasts the shock, HH only have a prior distribution. Then CB sets the policy rate i, and given this signal, HH make their expenditure decisions. In period 1 the shock occurs and HH set prices and consumption.

2.2 Equilibrium

The setup for the game is simplified by the fact that there is complete flexibility of prices in period 1 and all information is revealed in that period. Hence, no HH choice in that period affects the objective function of the CB in period 0. Also, since prices are completely rigid in period 0 and therefore each HH adjusts labor and output to meet demand for variety ℓ, the only relevant decision for HH is how much to consume in that period, which will be influenced by their belief about a future shock θ_1.[4]

The appendix develops a useful benchmark case where there is no uncertainty and there is full price flexibility in both periods 0 and 1. In such a model, agents first find out optimizing values in period 1 and work "backwards" in period 0 to decide on consumption in period 1. We can use the results derived in period 1 of that model for the current situation. Of course uncertainty needs to be accounted for, but this can be done easily by using contingent plans. The flexible price environment described in the appendix shows that HH choose $c_1^j = \theta_1$. Hence whatever the productivity turns out to be in period 1, HH will get utility $u(\theta_1)$. As of period 0, they expect utility to be $\mathbb{E}u(\theta_1)$ as stated in (2.1a).

Note that when CB maximizes (2.6) it has two channels by which it can change consumption in period 0. First, it can set the policy rate i as potentially different from the initial i_0 and second, it can influence HH's beliefs about θ_1, which through the expectation channel can also affect consumption c_0^j.

2.2.1 Perfect Bayesian Equilibrium

Restricting attention to pure-strategy Perfect Bayesian Equilibrium, agents' strategies are therefore defined as:

$$i \;:\; \theta_1 \mapsto \Re^+ \qquad \text{(CB, sender)} \tag{2.7a}$$

$$c_0^j \;:\; \Re^+ \mapsto \Re^+ \qquad \text{(HH, receiver)} \tag{2.7b}$$

Since CB perfectly forecasts θ_1, and HH observe the signal i prior to their consumption decisions, strategies are denoted according to (2.7) as $i(\theta_1)$ and $c_0^j(i)$ for the sender and the receiver respectively. These strategies form

[4]Note that the second equation in (2.4a) implies that by choosing consumption c_t^j and given $P_t(\ell)$, demand for variety ℓ by each HH is also determined.

a Perfect Bayesian Equilibrium if and only if:

$$i(\theta_1) = \arg\max_{i \in \Re^+} u(c_0) \tag{2.8a}$$

where given symmetry $c_0 = c_0^j$ and $u(c_0) = \int_0^1 u(c_0^j)dj$, and for any i and each agent (j):

$$c_0^j(i) \quad = \quad \arg\max_{c_0^j \in \Re^+} \left[u\left(c_0^j\right) + \beta \mathbb{E} u(\theta_1) \right] \tag{2.8b}$$

subject to constraints in (2.5). The expectation operator $\mathbb{E}$ refers to the Bayes posterior probability distribution, where μ is the probability that $\theta_1 = 1$, i.e., no contraction, that consumer j assigns having observed i:

$$\mu = Pr(\theta_1 = 1|i) = \frac{Pr(i|\theta_1 = 1)q}{Pr(i|\theta_1 = 1)q + Pr(i|\theta_1 = 1 - \Delta)(1-q)} \tag{2.9}$$

if $Pr(i|\theta_1 = 1)q + Pr(i|\theta_1 = 1 - \Delta)(1-q) > 0$.

2.2.2 Aggregate Consistency

The notion of aggregate consistency is modified from market clearing due to price rigidities in period 0. Nonetheless, the following conditions must be satisfied:

$$\text{Goods market:} \qquad c_t(\ell) \equiv \int_0^1 c_t^j(\ell)dj = y_t^\ell \tag{2.10a}$$

$$\text{Labor market:} \qquad \int_0^1 n_0^j dj \leq 1, \quad \int_0^1 n_1^j dj = 1 \tag{2.10b}$$

$$\text{Bonds market:} \qquad \int_0^1 B^j dj = 0 \tag{2.10c}$$

Equation (2.10a) states that the aggregate demand for variety ℓ, denoted $c_t(\ell)$, equals production of that variety. Note that in period 0 when prices are fixed, this implies that output will adjust to meet demand. Technologically, there is an upper bound on production, however, as $y_0^\ell = n_0^j$ and since HH are endowed with a unit of labor, the maximum amount of the good that can be produced is unity. Production may adjust downwards, however; if demand for variety ℓ decreases, output will meet this demand. This means that labor may fall below unity, as is expressed in (2.10b). If strict inequality holds, there are idle resources in the economy, which is a suboptimal situation because leisure is not valued by HH. We will informally refer to it as "unemployment". Finally, equation

(2.10c) states that the bond market must clear. In this case, since all agents are homogenous, this condition is satisfied with $B_i = 0$.

3 Analysis

Recall that when HH set prices initially they expect $\theta_1 = 1$. Equation (A.8) in the appendix shows that in such a flexible price environment the level of prices in period 0 will satisfy:

$$P_0(\ell) = P_0 = \frac{u'(\theta_0)}{\beta(1+i_0)u'(\theta_1)} = \frac{1}{\beta(1+i_0)} = \bar{P}, \tag{3.1}$$

because in this case $\theta_0 = \theta_1 = 1$ as perceived by HH. This is the level of prices for period 0 in place throughout the rest of the analysis and the level of prices introduced before in the budget constraint for period 0 in (2.5a). We present next the complete information benchmark where both CB and HH forecast the actual value of the future shock.

3.1 Complete Information Benchmark

Suppose that once HH set prices according to (3.1), they perfectly forecast the value of θ_1, just as the CB does. In this complete information case, the following Euler equation characterizes optimal consumption decisions for HH:[5]

$$u'(c_0^j(i)) = \beta\frac{(1+i)P_0}{P_1}u'(\theta_1) = \frac{1+i}{1+i_0}u'(\theta_1) \tag{3.2}$$

In deriving (3.2) we used the optimality conditions for problem (2.8b) under perfect foresight and the level of prices of (3.1). The second equality in the expression above also uses the assumption that money supply anchors $P_1 = 1$.

As is well known, given fixed prices monetary policy is powerful in inducing agents to modify current consumption due to substitution, because the real interest rate is affected by i. Nevertheless, we want to study a situation

[5]Again, we are using the result derived in the appendix that in a flexible price environment such as period 1, the actual consumption level for HH j satisfies: $c_1^j = \theta_1$.

where under the bad state $\theta_1 = 1-\Delta$, even the loosest monetary policy $i=0$ cannot restore "full employment," the situation where $n_0^j = 1$ for all j.

Let $c_0^j(i,\theta_1)$ denote the consumption level of HH j facing interest rate i and believing θ_1. Specifically, $c_0^j(i,1-\Delta)$ is consumption when facing i and believing a future contraction $\theta_1 = 1-\Delta$. And $c_0^j(i,1)$ is consumption when facing i and believing no future contraction $\theta_1 = 1$. The next proposition gives conditions under which the ZLB is reached.[6]

Proposition 1. The ZLB. *There exists a Δ^c satisfying:*

$$(1+i_0)u'(1) = u'(1-\Delta^c) \tag{3.3}$$

such that for all $\Delta > \Delta^c$, the ZLB $i(1-\Delta)=0$ cannot restore "full employment."

Proof. Assume that for $\Delta = \Delta^c$, $i(1-\Delta^c)=0$ and HH consumption satisfies $c_0^j(0,1-\Delta)=1$. In this case the Euler equation in (3.2) must be (3.3). That such a Δ exists follows from a simple application of the Intermediate Value Theorem. Let $f(\Delta) = u'(1-\Delta) - (1+i_0)u'(1)$, according to (2.1b) this function is continuous. Is easy to verify that it takes the following values:

$$\begin{aligned} f(0) &= u'(1) - (1+i_0)u'(1) = -i_0 u'(1) < 0 && (3.4)\\ f(1) &= \lim_{\Delta\to 1} u'(1-\Delta) - (1+i_0)u'(1) = +\infty && (3.5)\end{aligned}$$

Therefore, there exists $\Delta^c \in [0,1]$, such that (3.3) is satisfied.

It follows then that for all $\Delta > \Delta^c$, optimizing HH under the ZLB will satisfy the following Euler equation:

$$u'\left(c_0^j(0,1-\Delta)\right) = \frac{1}{1+i_0}u'\left(1-\Delta\right) > u'(1), \tag{3.6}$$

where the inequality at the end is satisfied because $u'\left(1-\Delta\right) > (1+i_0)u'(1)$, for any $\Delta > \Delta^c$, due to concavity of $u(\cdot)$. Concavity of $u(\cdot)$ also implies that

$$c_0^j(0,1-\Delta) < 1, \quad \Delta > \Delta^c. \tag{3.7}$$

[6]The notation for the level of consumption depending on beliefs is redundant in this section where there is complete information, but it will be useful later on.

$\square$

We have shown that if the ZLB is attained, and under a sufficiently large contraction, consumption falls below unity. This is undesirable since output can be produced without disutility from labor. To see the effect on labor formally, we look at market clearing (2.10c). Given homogeneity of HH, this condition delivers $B^j = 0$. Then from the budget constraint (2.5a) we get:

$$c_0^j(0, 1 - \Delta) = y_0^\ell = n_0^j(0, 1 - \Delta) < 1, \tag{3.8}$$

where, to be consistent with the notation for consumption, we label the resulting hours worked $n_0^j(i, \theta_1)$ when HH face i and expect θ_1.

For the remainder of this paper we will focus on this case where monetary policy is unable to restore first-best allocations. Hence we work with assumption 1 below.

Assumption 1. $\Delta > \Delta^c$.

Under assumption 1, "unemployment" is present and monetary policy is powerless to induce first-best allocations. A large enough contraction will push the economy to this ZLB. The initial i_0 plays a role here, since the lower this value is, the more likely that a given Δ will surpass Δ^c. A low initial nominal rate leaves "less room" to the CB to pursue full employment.

In the good state situation where $\theta_1 = 1$, CB does not need to change the policy rate from i_0 to improve welfare. The Euler equation from (3.2) is:

$$u'(c_0^j(i, 1)) = \frac{1 + i}{1 + i_0} u'(1). \tag{3.9}$$

Raising rates is detrimental as it would make agents want to save for the future. Lowering the rate from i_0 is of no use as the economy already operates at full employment. Then by leaving the interest rate at its initial level $i = i_0$, CB induces $c_0^j(i_0, 1) = 1$. Again, the budget constraint (2.5a) and market clearing (2.10c) imply that

$$c_0^j(i_0, 1) = y_0^\ell = n_0^j(i_0, 1) = 1, \tag{3.10}$$

hence, the following characterization emerges for agents' decisions in the complete information benchmark:

$$i(1) = i_0, \quad i(1-\Delta) = 0, \quad c_0^{1j}(i_0) = 1, \quad c_0^{\Delta j}(0) = [u'] \left(\frac{1}{1+i_0} u'(1-\Delta) \right)^{-1} \tag{3.11}$$

where the last equalities follow from (3.2) and $[u'](\cdot)^{-1}$ denote the inverse of $u'(\cdot)$. Due to symmetry among HH:

$$c_0^j(i_0, 1) = c_0(i, 1), \quad c_0^j(0, 1-\Delta) = c_0(0, 1-\Delta), \quad n_0^j(i_0, 1) = n_0(i_0, 1), \quad n_0^j(0, 1-\Delta) = n_0(0, 1-\Delta). \tag{3.12}$$

That is, aggregates equals individual values. Utility for HH and welfare, which CB aims to maximize is given by:

$$u(1) + \beta u(1), \quad u(c_0(0, 1-\Delta)) + \beta u(1-\Delta), \tag{3.13}$$

under the good state and the bad state respectively, where $u(1) > u(c_0(0, 1-\Delta))$.

3.2 The Incomplete Information Case

In the incomplete information case, the key equation to obtain HH's optimal response is the first-order condition for (2.8b):

$$u'(c_0^j(i)) = \frac{1+i}{1+i_0} \mathbb{E} u'(\theta_1), \tag{3.14}$$

where $\mathbb{E}$ refers to the expectation that uses (2.9).[7]

The first step in characterizing the sort of equilibria that may arise is to verify whether two conditions usually found in signaling games are satisfied: the existence of "envy" and the "single crossing" condition.

[7] Note that we are still assuming that CB commits to anchoring nominal values in period 1 in such a way that the price level is $P_1 = 1$.

3.2.1 Envy

Would a CB have an incentive to "masquerade" when forecasting a contraction? In other words, if CB, knowing that a contraction is coming, set:

$$i(1-\Delta) = i_0, \tag{3.15}$$

that is, the same policy rate it would set when forecasting no contraction in a complete information case, inducing HH to believe that no contraction is forecasted. Then equation (3.14) in its aggregate version becomes $u'(c_0(i_0, 1)) = u'(1)$. Which means, of course that $c_0(i_0, 1) = 1$, and maximal welfare is attained in period 0. This shows the power of beliefs induced by CB actions in this setup. Note also that, if assumption 1 is not satisfied, there is no reason for CB to be tempted to mislead HH.[8] Therefore, an interesting informational problem arise here under a special circumstance when the ZLB is attained.

It may seem striking at first that even though CB pursues maximization of social welfare, it has the incentive to mislead HH to believe that the economy would not suffer a contraction when it actually will. We want to dig into this feature by making reference to HH decisions about savings and the link that exists with production of varieties of goods in the economy. First, from the Euler equation (3.14) it is clear that if HH believe a contraction is coming, consumption demand in period 0, c_0^j, optimally decreases. We know that savings need to be zero in equilibrium $B^j = 0$. When prices are flexible, the desire to save decreases P_0, reducing the *real* rate and curbing HH decisions to save. Price flexibility allows the economy to optimally adjust. But when prices are rigid, the only way the real interest rate may fall is if the policy interest rate decreases. When the policy interest rate reaches the ZLB, no further decrease in the real interest rate is possible. Hence HH under the ZLB still want to save. Why does this translate into a suboptimal equilibrium? From the second equation in (2.4a), the aggregate demand of good ℓ can be derived:

$$c_0(\ell) \equiv \int_0^1 c_0^j(\ell) dj = \left(\frac{\bar{P}}{P_0}\right)^{-\eta} \int_0^1 c_0^j dj \equiv \left(\frac{\bar{P}}{P_0}\right)^{-\eta} c_0, \tag{3.16}$$

where $c_0(\ell)$ is defined as aggregate consumption demand of variety ℓ and $c_0 \equiv \int_0^1 c_0^j dj$ is aggregate consumption demand. When HH j decides to cut down on consumption c_0^j, all HH do the same and they cannot prevent a negative aggregate influence on c_0, which translates into reduced demand for the product each HH is producing

[8] Of course, if CB is forecasting no contraction, it has no incentive to go to the ZLB, thereby inducing HH to believe a recession is coming.

$c_0(\ell)$. This means that at given prices, demand for their product falls and they are forced to cut down on hours worked n_0^j even though utility does not fall with any feasible amount of hours worked.[9] This is illustrated in Figure 2. Point A represents a situation where HH, expecting $\theta_1 = 1$ and facing i_0, want to consume $c_0^j(i_0, 1)$.

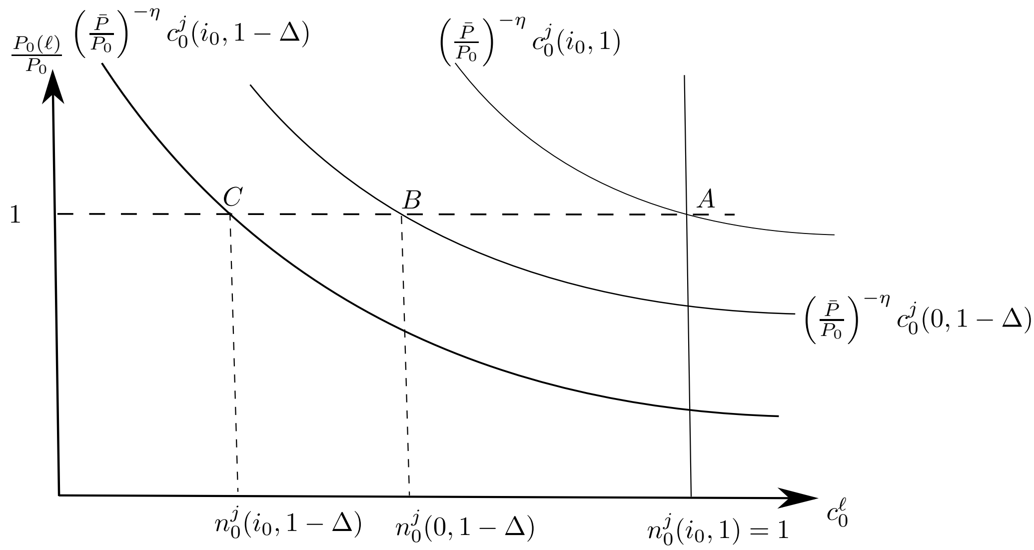


Figure 2: At point A all HH believe no contraction is coming and face i_0. Point C is a situation where, facing the same rate, HH believe a contraction is coming. At point B, HH believe a contraction is coming but they face the ZLB $i = 0$. "Unemployment," segment $A - B$, could be avoided if by setting $i = i_0$, CB induces HH to believe no contraction is coming.

Since demand for the product HH are producing does not fall, they work during the entire unit of time to meet that demand. If, facing the same interest rate, HH believe that $\theta_1 = 1 - \Delta$ then desired consumption is $c_0^j(i_0, 1-\Delta)$, which implies a low demand for the product they are producing. The outcome is point C where $n_0^j(i_0, 1-\Delta)$ is too low, which would lead to "unemployment" gap $A - C$. CB may improve things by lowering the interest rate down to the ZLB, reaching point B. Point A and point B are the equilibrium outcomes examined before in the complete information benchmark. If "envy" is present, CB forecasting a contraction would choose i_0, and *if* HH believe no contraction is coming, the absence of desire to save will prevent the economy from falling into a suboptimal situation of B. In this sense the unchanged policy rate serves as a coordination device that induces optimal allocations. Should HH believe in equilibrium that no contraction is coming when facing i_0? This is examined in the next section. Note however that if HH face i_0 and believe that $\theta_1 = 1 - \Delta$, the worst possible case would arise as reduced demand would make C the equilibrium situation.

[9]There is a "coordination failure" among HH. If they could somehow coordinate to not reduce c_0^j, they could avoid ending up with idle resources. Coordination failures in Keynesian models were analyzed in general terms by Cooper and John [1988].

3.2.2 Single Crossing Condition (SCC)

If the SCC holds then it should be more costly for a CB forecasting a contraction to *increase* the policy interest rate than for a CB not forecasting a contraction. But the policy interest rate by itself does not have an impact on welfare, only on expectations. Therefore for a fixed level of consumption, the policy rate has no effect on utility:

$$\left.\frac{\partial u(c_0)}{\partial i}\right|_{\theta_1=1} = \left.\frac{\partial u(c_0)}{\partial i}\right|_{\theta_1=1-\Delta} = 0. \tag{3.17}$$

The macroeconomic model therefore does not support the SCC and the existence of a separating equilibrium can thus be ruled out. We are left to examine whether a pooling equilibrium may arise.

For a SCC to arise in this setup, welfare must be affected *directly* by the interest rate, which is not the case in this model, or in any standard monetary policy model for that matter. The only effect that the interest rate has is on consumption, in the "response to the signal" in the game theoretic jargon. This feature would be maintained under fairly general conditions, any change in the decision variable for HH would be a response to the signal, a response to the policy rate i.[10]

3.2.3 Pooling Equilibrium

Under a pooling equilibrium the two types of CB chooses the same interest rate i^p, that is, independently of the value forecasted for θ_1. Given that HH would observe a unique interest rate regardless of whether CB foresees a contraction or not, their Bayesian posterior belief remains equal to the prior in (2.3).[11]

Proposition 2. Pooling equilibria. *Let i^s be such that:*

$$u(c_0(0, 1-\Delta)) = u(c_0(i^s, 1)), \tag{3.18a}$$

that is, the value of the interest rate that would equate period 0 utility for HH believing no contraction is coming with utility under the ZLB when HH believe a contraction is coming. For some $q \geq q^c$, there exists a set of

[10]For example, if costly price changes are introduced, then HH would like to change prices when facing a policy rate i different than i_0, but there would not be an independent effect of the policy rate on HH's welfare.

[11]From (2.9): $\mu = \frac{Pr(i|\theta_1=1)q}{Pr(i|\theta_1=1)q+Pr(i|\theta_1=1-\Delta)(1-q)}$. In this case upon observing i^p, HH assign $Pr(i^p|\theta_1 = 1) = 1 = Pr(i^p|\theta_1 = 1-\Delta)$, and hence $\mu = Pr(\theta_1 = 1|i^p) = q$.

pooling equilibria $\mathcal{P} = [0, i^s]$, *where a CB, independently of the forecasted value of* θ_1, *chooses* $i^p \in \mathcal{P}$. *HH* beliefs *are given by:*

$$\mu = \begin{cases} q & \text{if} \quad i = i^p \\ 0 & \text{if} \quad i \neq i^p, \end{cases} \tag{3.18b}$$

where the first line in (3.18b) *corresponds to the on-equilibrium beliefs derived from Bayes' rule and the second line corresponds to the out-of-equilibrium beliefs, which are unconstrained by BayesâĂŹs law and are defined by assumption.*[12]

The resulting consumption and its utility in period 0 are given by $u(c_0(i^p, q)) \in [u(c_0(0, 1-\Delta)), u(1)]$. $c_0(i^p, q)$ *denotes consumption upon observing the pooling policy rate* i^p, *and when HH's posterior beliefs remain equal to their prior* (2.3), *where* q *is the exogenous probability of no contraction.* q^c *is given by:*

$$q^c \equiv \frac{i^p}{1+i^p} \frac{u'(1-\Delta)}{u'(1-\Delta) - u'(1)} \tag{3.18c}$$

Proof. Out-of-equilibrium beliefs induce CB to have precisely the ZLB $i = 0$ as the most favorable deviation from i^p. If HH believe that a contraction is coming, CB optimally goes to the ZLB. For a pooling equilibrium to arise, it must be the case that:

$$u\left(c_0(0, 1-\Delta)\right) \leq u(c_0(i^p, q)) \tag{3.19a}$$

where: and $c_0(i^p, q)$, according to the Euler equation derived from the aggregate version in (2.8b), satisfies:

$$u'\left(c_0(i^p, q)\right) \geq \frac{1+i^p}{1+i_0} \left[q u'(1) + (1-q) u'(1-\Delta)\right] \tag{3.19b}$$

with $(>)$ when $c_0(i^p, q) = 1$. Note that inequality (3.19a) implies:

$$u'(c_0(0, 1-\Delta)) \geq u'(c_0(i^p, q)). \tag{3.20a}$$

[12] Note that the out-of-equilibrium beliefs cannot be refined by means of dominance-based refinements such as the dominance criterion and the dominance-in-equilibrium criterion as in the "intuitive criterion" of Cho and Kreps [1987] because both CB types in our model share the same utility function.

Euler's equation (3.2) also implies:

$$u'\left(c_0(0,1-\Delta)\right) \quad = \quad \frac{1}{1+i_0}u'(1-\Delta), \tag{3.20b}$$

that we know is satisfied with equality as under assumption 1 consumption is suboptimal, below unity. Using (3.20b) and (3.19b) in (3.20a):

$$\frac{1+i^p}{1+i_0}\left[qu'(1)+(1-q)u'(1-\Delta)\right] \leq \frac{1}{1+i_0}u'(1-\Delta), \tag{3.20c}$$

from which we can find out the value of q^c for given i^p, Δ:

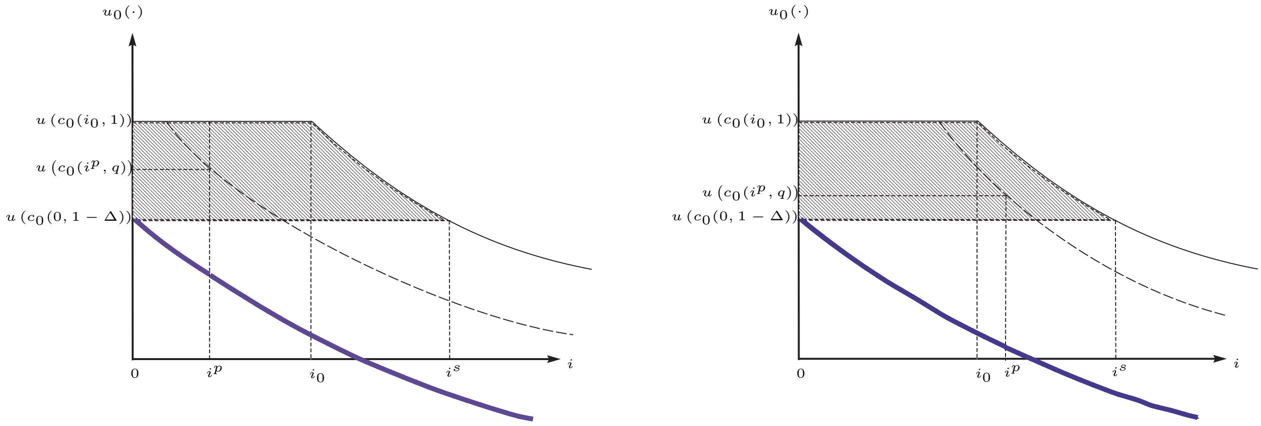
$$q \geq \frac{i^p}{1+i^p}\frac{u'(1-\Delta)}{u'(1-\Delta)-u'(1)} \equiv q^c. \tag{3.20d}$$

Clearly q^c is above 0. To show that it is below unity, by way of contradiction, assume that:

$$(1+i^p)u'(1) > u'(1-\Delta) \tag{3.20e}$$

Let us show that inequality (3.20e) cannot be satisfied for any $i^p \in \mathcal{P}$. Note that the LHS of (3.20e) linearly increase with i^p. First, for $i^p = 0$, the contradiction is immediate. Second, for $i^p = i^s$, by definition of i^s and Euler's equation we have $u'(1-\Delta) = (1+i^s)u'(1)$. A contradiction. $\square$

Equilibria is described in Figure 3. We discuss two cases, separately, the case when $i^p \leq i_0$ and the case when $i^p > i_0$. The dotted curve in both figures is given by utility of consumption $u(c_0(i))$ assuming that HH have $\mu = q$. Of course only at $i = i^p$ is that conjecture validated by Bayes law in equilibrium. In panel(a) of the figure, i^p is the pooling equilibrium. Independently of the CB forecast, CB will choose that rate. If CB forecasts a future contraction, i^p delivers higher utility than going to ZLB (a situation arising under the complete information situation). If CB forecasts no future contraction, it is worse off compared to the complete information situation. A similar configuration arise in panel (b) of the figure. There, i^p is higher than the base rate i_0. This is however counterintuitive as an equilibrium, since the news of a possible contraction induce the CB to increase the policy rate. Given out-of-equilibrium beliefs in (3.18b), nothing prevents this to happen in the model.



(a) $i^p \leq i_0$ (b) $i^p > i_0$

Figure 3: Pooling equilibria: Independently of the CB forecast, it sets $i = i^p$. The black dotted line is welfare for different values of i *if* HH regard $\mu = q$. Only at i^p is this conjecture validated as an equilibrium. For the rest of the policy rates, welfare is given by the blue thick line. For high enough values of q, given the same i^p, welfare is $u(1)$, the maximum possible. For the stipulated out-of-equilibrium beliefs, the whole set in gray represents pooling equilibria outcomes for some (q, i^p).

3.2.4 Discussion about the Pooling Equilibria

It is instructive to find conditions under which a given belief $\mu = q$, pooling equilibria may or may not arise. It is straightforward to verify from (3.20d) that:

$$\frac{\partial q^c}{\partial i^p} > 0, \quad \frac{\partial q^c}{\partial \Delta} < 0. \tag{3.21}$$

Hence while generally the existence of a pooling equilibrium requires high enough prior beliefs that no contraction is coming, the higher i^p is given a possible contraction, the smaller the set of pooling equilibria. Also, the second inequality in (3.21) show that the higher the contraction Δ, the larger is the set of pooling equilibria. The set of pooling equilibria is larger when the expected contraction is severe; this represents the willingness of the CB to pool and not induce agents to believe a grim scenario is expected when actually it may well happen that CB is not forecasting a contraction at all. In this situation, the CB is most likely to distort its complete information policy interest rate, either by reducing it or increasing it beyond i_0. This last case also implies that the CB is willing to induce a contraction by *increasing* rates when privately forecasting no contraction. While welfare in this case is lower than the complete information counterpart, CB optimally does this to avoid sending a wrong signal to HH that the economy will suffer an adverse shock in the future.

As illustrated in Figure 3, there is a wide set of pooling equilibria, corresponding to the gray area. Depending on the pooling equilibrium selected, the resulting welfare ranges from the lowest possible under the ZLB to the same welfare as if the economy were not to undergo a contraction, even if it actually does. Multiplicity of equilibria also means of course that there are multiple predictions for equilibrium "unemployment" ranging from no unemployment to the same level of unemployment as under the ZLB.

If the CB forecasts a contraction, generally it will gain by pooling, because by choosing a higher rate than the ZLB it will successfully induce HH to assign a positive probability of no contraction, leading them to only partially reduce consumption. Note that in the gray area above i_0 this pooling equilibrium implies that even though the policy rate is high, consumption is above the minimum possible, which occurs under a contraction and at the ZLB. Two opposing effects are in place here. First, due to a traditional transmission mechanism, consumption tends to decline with the high rate. Second, the expectation channel induces HH to believe that no contraction is coming and they end up with large consumption and welfare. This shows the power of the "expectations channel" in this signaling game.

3.2.5 Robustness

We now explore the robustness of the results found to alternative of out-of-equilibrium beliefs. While these beliefs cannot be refined in our model by standard refinement criteria because utility functions of the two types of CB share the same preferences, we rule out implausible equilibria by constraining the set of out-of-equilibrium beliefs.

Out-of-equilibrium beliefs (3.18b) are not constrained by the definition of equilibrium itself but it may occur that some other beliefs are more reasonable or appealing. For example, for equilibria in the area above i_0 it is counterintuitive that HH's out-of-equilibrium beliefs assign 0 probability to no contraction when observing an increase in the policy interest rate. We do not observe such equilibria in reality. When there are news of a possible recession, CBs usually conduct expansionary policy, not contractive. To obtain a simple characterization, we assume that if HH observed a rate greater or equal than i_0 they would hold out-of-equilibrium beliefs equal to the prior (2.3).

$$\mu = \begin{cases} q & \text{if} \quad i = i^p \\ 0 & \text{if} \quad i_0 > i \neq i^p \\ q & \text{if} \quad i_0 \leq i \neq i^p \end{cases} \tag{3.22}$$

We show that under these beliefs there cannot be pooling equilibria above i_0. Note that for potential i^p in this region, the most favorable deviation is i_0. Therefore, by way of contradiction, assume that $i^p > i_0$ is a pooling equilibrium. Then: $u(c_0(i_0, q)) < u(c_0(i^p, q))$. Concavity implies $u'(c_0(i_0, q)) > u'(c_0(i^p, q))$. By the Euler equation:

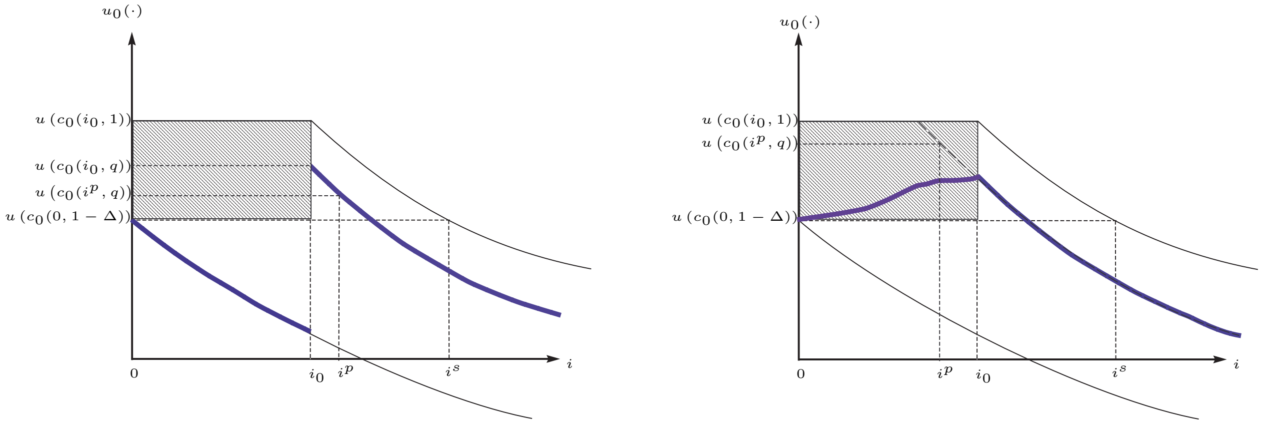
$$(1 + i_0)[qu'(1) + (1 - q)u'(1 - \Delta)] > (1 + i^p)[qu'(1) + (1 - q)u'(1 - \Delta)] \tag{3.23}$$

which gives an immediate contradiction. This can be easily seen in panel (a) of Figure 4. i^p cannot be an equilibrium there, since by decreasing the policy interest rate, utility increases along the thick blue curve. By setting i_0 utility is maximal independent of the forecasted value of θ_1, and utility obtained is $u(c_0(i_0, q)) > u(c_0(i_p, q))$. The same is true for any pooling equilibria in the region above i_0.

As an additional robustness check, consider the case when $i < i_0$. Out-of-equilibrium beliefs in either (3.18b) of (3.22) imply that a deviation from the pooling equilibrium induces HH to believe that a contraction is coming with probability one. We assume here that out-of-equilibrium beliefs are an increasing monotone function of i, $\psi(i)$. We impose the conditions $\psi(0) = 0$ and $\psi(i_0) = q$, that is, when observing the ZLB HH believe that a contraction is coming for sure and when observing i_0 they stick to their prior belief. This relaxes the previous assumption significantly, especially for large q since in this case the higher the interest rate, the larger is the probability that HH assign to no contraction. To see this, let us define formally:

$$\mu = \begin{cases} q & \text{if} \quad i = i^p \\ \psi(i) & \text{if} \quad i_0 > i \neq i^p \\ q & \text{if} \quad i_0 \leq i \neq i^p \end{cases} \tag{3.24}$$

Hence we are still maintaining the same out-of-equilibrium beliefs for $i \geq i_0$ as in (3.22). To characterize equilibria consider panel (b) of Figure 4 where q is relatively high. The black dotted curve shows utility for different values of i when HH expect no contraction with probability q. i^p is a pooling equilibrium because any deviation yields



(a) μ given by (3.22) (b) μ given by (3.24)

Figure 4: In panel (a), given the out-of-equilibrium beliefs portrayed in the thick line i^p cannot be an equilibrium because CB can reduce the interest rate up to i_0 and obtain higher utility. In panel (b) when $i < i_0$ out-of-equilibrium beliefs are described by $\psi(i)$ (assumed to be monotone increasing and satisfying the boundary conditions explained above. i^p is an equilibrium for the depicted q as no deviation is profitable for the CB.

lower utility given by the blue thick curve and the corresponding out-of-equilibrium-beliefs, and therefore both CB types are better-off by choosing i^p, more so the CB forecasting a contraction. More formally, note that for such an example, the most favorable deviation is i_0. Therefore for i^p to be an equilibrium, it has to be the case that $u(c_0(i_0, q)) < u(c_0(i^p, q))$, and concavity gives $u'(c_0(i_0, q)) > u'(c_0(i^p, q))$. The Euler equation then gives expression (3.23), which is obviously satisfied because in this case $i^p < i_0$.

By varying $q \in [0, 1]$ it is evident that the gray area depicts possible pooling equilibria. Hence, out-of-equilibrium beliefs need not be as pessimistic as in (3.18b) for allocations in the relevant gray area to be sustained as possible equilibria.

3.3 The Symmetric Incomplete Information Case

A crucial assumption driving the previous results is that CB is better informed about future shocks hitting the economy than households. We now present a benchmark case where the CB and households are symmetrically informed about future shocks. We model this benchmark by assuming that the CB does not perfectly forecast θ_1, but instead hold the same beliefs about the probability of future shocks than HH, namely (2.3).

In this case, in period 0 CB sets i_0 expecting no contraction and HH set prices. Later on during the first period, *both* CB and HH receive news of a possible future contraction according to (2.3). The Euler equation for HH in its aggregate version is:

$$u'(c_0(i)) \geq \frac{1+i}{1+i_0}\left[qu'(1) + (1-q)u'(1-\Delta)\right] \tag{3.25}$$

with ($>$) when $c_0(i) = 1$, for some i. Now the consumption-maximizing CB is not constrained by signaling considerations, and therefore it can increase consumption and welfare by further reductions in interest rates until reaching either a zero interest rate or full employment (which would depend on the exogenous value of q). For example, in panel (b) of Figure 4 by reducing the interest rate below i^p, absent signaling considerations, the CB would increase utility unambiguously, and both CB and HH are better-off reaching utility level $u(c_0(i_0, 1))$, as if the economy were not hit by a contractive shock. Since CB does not have an informational advantage over households, HH would not interpret a lower interest rate as a signal of a likelier contraction.

This benchmark suggests that the CB would like to avoid being informed about future shock if given the chance. This situation arise not because the CB does not value information per se, but by being informed about the future opens the possibility that HH believe that CB will try to conceal some information, which we have shown delivers the suboptimal pooling equilibria.

All other things held constant, the equilibrium interest rate would be lower that would be under a pooling equilibrium (for a given exogenous value of q), which reinforces the main result of the previous section, namely that a better informed CB would be more cautious about reducing the interest rate in order to avoid signaling to households the possibility of an adverse shock in the future. However, we claim that the incomplete information benchmark is a more relevant case because the assumption that CBs are better informed than HH (and HH also believe that) are generally supported by the literature.[13]

4 Conclusions

This paper explores the implications of asymmetric information about the future status of the economy between a CB and private agents when the former conducts monetary policy near the zero lower bound. The main finding is that there is multiplicity of pooling equilibria in which the policy interest rate is above the zero lower

[13] See for example Romer and Romer [2000], Peek et al. [2003], Hubert [2015] and Pedersen [2015].

bound. Our results suggest that a CB privately foreseeing a recession will follow a less expansionary monetary policy compared to either a complete information or a symmetric information context in order to avoid making matters worse by revealing bad times ahead, which would further decrease private expenditure and deepen the contraction. In such an equilibrium the CB that does not foresee a contraction complies with the pooling equilibrium policy rate, which is welfare detrimental compared to a complete information situation.

Our results are consistent with stylized facts in the actual conduct of monetary policy. In particular, there is evidence that when conducting expansionary monetary policy in difficult times CBs tend to cut rates in a very prudent manner, preferring a sequence of minor adjustments over time rather than large ones, unless it is very evident that the economy is in recession. This is consistent with our equilibria where the policy rate is set above the zero lower bound when the prior belief of no future contraction is relatively high. This means that an equilibrium policy rate above the zero lower bound under an actual future contraction is less likely if it is quite evident to all market participants that a contraction will occur. This would have been the case, for example, in the most recent global financial crisis where CBs around the world indeed cut rates to the zero lower bound quite quickly. Yet in less difficult times, usually in the early stages of a deep contraction, is not uncommon to observe CBs being careful not to induce "panic" about the future state of the economy by cutting rates aggressively. It is also fair to say that prudent behavior by CBs is consistent also with other models that emphasize CBs own struggle in acquiring better information to make decisions. See for example Aoki [2003] and Gust et al. [2015] for the case of the ZLB. Testing empirically both alternative and observationally similar theories explaining CBs' prudence in reducing interest rates when facing contractions remains an interesting empirical question open for future research. However, what is unique about our model is the result that even a CB anticipating no contractions in the future would distort its interest rate policy (at a welfare cost) to prevent an adverse interpretation about the future of the economy by consumers.

In our analysis we maintained the strong assumption that prices are completely rigid. Also, the nature of the shock analyzed is very specific: It is a future productivity shock that leads to a current demand contraction. This implies that CB faces no trade-off for monetary policy, even if costly price changes would be allowed. It might be interesting to analyze situations where CB faces a supply contraction, if prices are allowed to change, this may deliver an interesting configuration for the signaling channel. This venue of research is left for future work.

Another possible interesting extension is to characterize the signaling game under more possible shocks or even a continuum of future shocks, which is often the case in New Keynesian models. In this paper, for tractability we considered two states of a future shock. However, we claim this assumption may be also justified because usually expectations about "future economic conditions" and "confidence climate" of firms and households are in practice framed and communicated in a simpler binary or finite-state setting, not in a continuous setting. Our setting is thus more consistent with this empirical way of framing and interpreting information about future economic conditions, see for example OECD [2003].

A Appendix

In this appendix I study a flexible price economy, where the CB maintains i_0 throughout period 0 and there is no uncertainty, with θ_0 and θ_1 being the productivity levels for periods 0 and 1 respectively.

Environment

As in the main model, households are indexed by j and each produce a given variety ℓ. Taking as given P_t and i_0, HH maximize:

$$u^j = u\left(c_0^j\right) + \beta u\left(c_1^j\right) \tag{A.1}$$

subject to:

$$P_0 c_0^j + B^j = P_0(\ell) y_0^\ell, \quad y_0^j = \theta_0 n_0^j \tag{A.2a}$$

$$P_1 c_1^j = (1+i_0)B^j + P_1(\ell) y_1^\ell, \quad y_1^j = \theta_1 n_1^j. \tag{A.2b}$$

Choosing $P_t(\ell), c_t^j, y_t^\ell$ and n_t^j, where the price level and demand for variety ℓ are given by:

$$P_t = \left[\int_0^1 P_t(\ell)^{1-\eta} d\ell\right]^{\frac{1}{1-\eta}}, \quad c_t^j(\ell) = \left(\frac{P_t(\ell)}{P_t}\right)^{-\eta} c_t^j \tag{A.3a}$$

where $c_t^j(\ell)$ is demand of variety ℓ by HH j, money is demanded each period for transaction purposes:

$$P_t c_t^j = M_t^j \tag{A.3b}$$

Definition of Equilibrium

A (monopolistic) competitive equilibrium is a price level P_t and an interest rate i_0 such that:

- *HH maximize utility* (A.1) *subject to the constraints* (A.2)

- *Markets clear:*

$$\textit{Goods market clears:} \qquad c_t(\ell) \equiv \int_0^1 c_t^j(\ell)dj = y_t^\ell \tag{A.4a}$$

$$\textit{Bonds market clears:} \qquad \int_0^1 B^j dj = 0 \tag{A.4b}$$

$$\textit{Money market clears:} \qquad \int_0^1 M_t^j dj = M_t^s \tag{A.4c}$$

Solution

The intra-temporal problem of how to set prices can be written for HH as maximizing real income from the production of variety ℓ:

$$\max_{P_t(\ell), n_t^\ell} \frac{P_t(\ell)}{P_t} y_t^\ell \tag{A.5a}$$

subject to:

$$y_t^\ell = c_t(\ell), \quad y_t^\ell = \theta_t n_t^j, \quad 0 \leq n_t^j \leq 1. \tag{A.5b}$$

where $c_t(\ell)$ is the market demand for variety ℓ:

$$c_t(\ell) \equiv \int_0^1 c_t^j(\ell)dj = \left(\frac{P_t(\ell)}{P_t}\right)^{-\eta} \int_0^1 c_t^j dj \equiv \left(\frac{P_t(\ell)}{P_t}\right)^{-\eta} c_t \tag{A.5c}$$

and $\int_0^1 c_t^j dj \equiv c_t$ is defined as aggregate consumption.

Using (A.5c), HH's problem can be written as:

$$\max_{P_t(\ell)} \left(\frac{P_t(\ell)}{P_t}\right)^{1-\eta} c_t, \text{ subject to: } n_t^j \equiv \left(\frac{P_t(\ell)}{P_t}\right)^{-\eta} \frac{c_t}{\theta_t} \leq 1 \tag{A.5d}$$

Let λ_t be the multiplier for the constraint in (A.5d). The K-K-T conditions are:

$$\frac{P_t(\ell)}{P_t} = \frac{\eta}{\eta - 1} \frac{\lambda_t}{\theta_t}, \quad \lambda_t \left[1 - \left(\frac{P_t(\ell)}{P_t}\right)^{-\eta} \frac{c_t}{\theta_t}\right], \quad \lambda_t \geq 0 \tag{A.5e}$$

By way of contradiction it is straightforward to show that $n_t^j < 1$ cannot be optimal. If $\lambda_t = 0$ then $P_t(\ell) = 0$

but the iso-elastic demand for ℓ implies that demand for product ℓ is infinite at that price. This clearly violates the restriction that $n_t^j \leq 1$, hence $n_t^j = 1$. Then with flexibility of prices $P_t(\ell) = P_t$, and market clearing in the bond market implies $c_t = \theta_t$. It is also immediate that $c_t = \theta_t = y_t^\ell = c_t(\ell)$.

As for the intertemporal consumption decision, the Euler equation in its aggregate form is:

$$u'(\theta_0) = \beta(1 + i_0)\frac{P_0}{P_1}u'(\theta_1) \tag{A.6}$$

Both in period 0 and in period 1, money market clearing should satisfy:

$$\int_0^1 M_t^j dj = P_t\theta_t = M_t^s \tag{A.7}$$

where M_t^s is the stock of money supply. The stock of money anchors the prices $P_1 = 1$, by assumption. In period 0 prices are determined by the Euler equation:

$$P_0 = \frac{u'(\theta_0)}{\beta(1 + i_0)u'(\theta_1)} \tag{A.8}$$

And given the interest rate i_0, money supply in period 0 adjusts to clear the money market, using (A.7):

$$M_0^s = \frac{\theta_0 u'(\theta_0)}{\beta(1 + i_0)u'(\theta_1)} \tag{A.9}$$

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