ESCUELA DE NEGOCIOS Y ECONOMÍA

PONTIFICIA UNIVERSIDAD CATÓLICA DE VALPARAÍSO

Working Papers

2020-01

Financial Deepening, Credit Crises, Human Capital and Growth

Sergio Salas

Pontificia Universidad Católica de Valparaíso and Dominican University

Kathleen Odell

Dominican University

ene.pucv.cl/wp

Financial Deepening, Credit Crises, Human Capital and Growth

Sergio Salas [†] Kathleen Odell [*]

May, 2020

Abstract

In spite of extensive research exploring the implications of financial matters for economic growth, a general equilibrium macroeconomic model of financial frictions with human capital as an engine of growth is lacking in the literature. This paper helps to fill this gap, proposing a model that includes endogenous growth, human capital, and financial constraints. We derive short-term and long-term predictions from the model. From a long run perspective, we explore the relationship between financial depth and growth, and predict that this relationship is non-monotonic. Higher financial depth is initially associated with higher growth, but at diminishing rates. Further increases in financial depth become growth detrimental. From a short-run perspective, we analyze the role of transitory financial disruptions in producing persistent economic changes, a phenomenon that arguably happened during the Great Recession and the years that followed. We propose an explanation for these persistent effects based on human capital.

Keywords: endogenous growth, financial depth, credit crunch, human capital, heterogeneous agents, fiscal policy.

JEL Classification: O4, E44.

[†]Dominican University and Pontifical Catholic University of Valparaiso. I would like to thank Dominican University where this paper is being developed as part of the *Norman and Ruth Carroll Endowed Chair in Business and Economics* of which I am the inaugural Chair. Email: sergiosalaslan@gmail.com

[*]Dominican University. Email: kodell@dom.edu

1 Introduction

In spite of a large body of research regarding the implications of financial matters for economic growth, a general equilibrium macroeconomic model of financial frictions that includes human capital as an engine of growth is still lacking in the literature. This situation is particularly perplexing because at least since Lucas [1998] and Romer [1990] human capital has been proposed as a key determinant of growth, and the empirical evidence supporting this proposal has led to widespread consensus regarding human capital's importance in the growth process. This paper intends to help fill this gap by proposing a model that includes all three elements: endogenous growth, human capital, and financial constraints. We use the model to study two main themes. First, we explore the relationship between finance and growth, where previous literature has found mixed results. We contribute from a theoretical perspective, assessing if such a relationship could be non-monotonic, something found in several recent empirical studies. Second, we examine the role of transitory financial disruptions in producing persistent changes in the economy. Such a persistent change arguably happened in the US following the Great Recession; we propose an explanation for this based on the human capital channel.

To provide insight into both the long-run and short-run questions described above, we develop a model that is composed of a continuum of infinitely lived households that decide on accumulation of physical and human capital, which are both rented as inputs of a production function run by a CRS firm. The firm therefore is modeled in simple terms, but this serves our purposes because the assumption that both factors of production can be accumulated gives us perpetual endogenous growth as diminishing returns are eliminated. Credit arises endogenously in the economy based upon the assumption that households face idiosyncratic preference shocks –a fiction that introduces liquidity needs– and frictions in trading resulting from the timing of actions they can take. We require households to make decisions about their capital accumulation (both physical and human) at the beginning of each period, prior to knowing the nature of a preference shock to be experienced later in the period. This decision-making timing structure is reasonable given that usually, capital investment decisions are long-term decisions and at least partially illiquid. Once the shock is realized, households cannot modify their capital accumulation decisions but they can resort to a credit market where bonds are traded to either finance more consumption than current income permits or to save for the next period. While households can resort to the credit market to finance their consumption needs; they face a borrowing constraint modeled as an exogenous fraction of their wealth. Since the preference shock is idiosyncratic, households will have different histories and different patterns of consumption, capital accumulation and savings over time regardless of initial endowments. One of the virtues of the model is that in spite of its relative complexity, suitable assumptions that do not sacrifice generality deliver simple aggregation, and we do not need to keep track of distributions.

Regarding the relationship between finance and growth, we study the long-run macroeconomic implications of relaxing the borrowing constraint. In so doing, we derive several testable implications of the model, allowing us to explore the relationship between financial depth and several other macroeconomic variables including the economy's rate of growth. Interestingly, we find that under some plausible parameterizations of the model, a non-monotonic relationship arises among financial depth and long run growth. Specifically, departing from a rough calibration of the model, further relaxations of the borrowing constraint would increase growth but at diminishing rates, and eventually further financial deepening may decrease growth.

Our second use of the model, exploring the persistent effects of transitory financial disruptions, is motivated by evidence that since the end of the Great Recession of 2008, the US economy has not recovered as strongly as needed to return GDP to its previous trend. Could a transitory credit crunch – a temporary tightening of the borrowing constraint– produce a recession and a permanent downward shift in GDP's trend? We find that under some empirically reasonable parameterizations of the model, the answer is yes. An intuition about why a transitory credit disruption produces a permanent break in GDP's trend is that knowing that credit will be more difficult to obtain in the future, households –facing the unchanged likelihood of a liquidity need– consider the investment in illiquid physical and human capital to be more costly. They therefore cut down on investments, which in turn harms growth and negatively affects the trend at which GDP was growing. At the same time, the crunch also produces a decrease in the interest rate, which may incentivize savers to invest in capital accumulation rather than deferring consumption though savings. These opposing effects on capital investments are related to income and substitution effects, but we show that under empirically reasonable parameterizations of the utility function, the net effect is a decrease in capital accumulation, producing the negative effect on GDP's trend. Since the result hinges on the negative effect of the credit crunch on human capital accumulation, we discuss in the text the plausibility of human capital decreases during and after the Great Recession. While direct evidence is limited, we argue that general human capital decay during the Great Recession cannot be ruled out.[1] As an extension, we also analyze the effectiveness of fiscal policy in mitigating the negative effects of the credit crunch. We now turn to describe how our contribution is situated within the existing literature.

Relationship with the existing literature

There is a robust body of empirical literature exploring the relationship between financial markets and economic growth, dating back to King and Levine [1993]'s findings of a positive relationship between the two. Many subsequent studies have found a similar positive association between growth and financial markets. Summarizing this literature, Levine [2005] concludes

[1] In section 4, specifically in subsection 4.2 we review some data and literature supporting the notion that human capital accumulation may have been negatively affected during the Great Recession. Viewing human capital even more broadly as including *intangible* capital gives also another perspective on the notion that a financial disruption could have affected capital accumulation. Corrado et al. [2009] discuss the importance of intangible capital for growth in the U.S.

that while there is evidence that financial development matters for growth, this is "subject to ample qualifications and countervailing views." The more recent empirical literature shows mixed results regarding this relationship, with some studies finding a threshold beyond which the relationship between financial development and growth changes from positive (as it has historically been understood), to neutral or even negative. Arcand et al. [2015] present empirical evidence that financial deepening becomes associated with negative growth once private credit reaches 100% of GDP. Law and Singh [2014] similarly identify a threshold beyond which further financial development harms growth. Other studies identify important differences in the growth effects of different types of credit. For example, Beck et al. [2012] compare the growth effects of enterprise versus household lending, and find the for the 45 developed and developing countries within their sample, enterprise lending is associated with growth while household lending is not. Rousseau and Wachtel [2009] show that the positive relationship between finance and growth that is clear in data from the 1960s to the 1980s is not observed in more recent data. These authors explore various explanations for the difference over time, including financial crises and the growth of equity markets; overall they conclude that there is substantial complexity in the finance growth relationship, especially in the last two decades. Despite the extensive research in this area, the nature of the finance/growth relationship is not yet fully understood.

While the empirical literature concerning finance and growth is extensive (if inconclusive), there are very few empirical studies in this area that also consider the role of human capital. This is a surprising gap in the literature, as human capital almost certainly matters for growth, and there are a number of clear pathways through which finance and human capital accumulation are likely to be related.[2] One early empirical study of the relationship between these three (finance, human capital, and growth) is Evans et al. [2002]. This paper examines 82 countries over a

[2]Lochner and Monge-Naranjo [2011] model this relationship in the US under imperfect credit markets based upon the structure of government student loan programs. Lochner and Monge-Naranjo [2012] provide a review of the evidence on the credit/education relationship. Caucutt and Lochner [2020] provide an exploration of the role of credit constraints in lifetime human capital accumulation.

period of 21 years, and finds that financial development is as important as human capital as a determinant of growth. In addition, these authors find that the interaction between credit and human capital makes a significant contribution to growth; specifically, this paper argues that a developed financial system is an essential complement to investment in human capital. A similar approach is employed in Hakeem [2010] with a focus on Sub-Saharan Africa. This paper finds that both human and physical capital are important to growth, while financial development is not shown to have a direct effect. However, the author identifies important interaction effects (complementarity) between financial development and human capital development, suggesting that simultaneous investment in both areas is growth enhancing. Abubakar et al. [2015] studies the impact of both bank and domestic private credit both on growth, both directly and indirectly through the human capital channel, in the Economic Community of West African States for the period 1980-2011. Results show that developing regional credit markets will ease credit constraints and encourage human capital accumulation, which will subsequently increase the growth of real GDP. The authors argue that empowering human capital investment through private credit is an important economic development strategy for the region. Kilic and Ozcan [2018] conduct a similar study focused determining the effect of financial development on human capital in Emerging Market Economies from 1990-2015; these authors find that human capital investment leads to more developed financial markets, but they do not find causality running from financial markets to human capital investment. Oyinlola and Adedeji [2019] study the role of financial development in the human capital/growth relationship in Sub-Saharan Africa from 1999 to 2014, concluding that deepening and enhancing the efficiency of the financial sector is likely to have spillover effects on human capital development. Overall, the emerging body of empirical literature is small but suggests that finance, human capital and growth are related in ways that are not yet fully understood.[3]

[3]Notably, many of the empirical papers cited (with the exception of Evans et al. [2002]) are focused on developing economies.

There is also a small body of theoretical work exploring the relationship between financial markets, human capital, and growth. One very recent paper that explores the relationship between the three is Nguyen [2019], which focuses on the intersection of financial deepening and the growth maximizing tax rate. Another contribution is Mino [2015], who actually considers a model with endogenous growth and financial frictions, deriving results for the balanced growth path. But this paper does not tackle any of the issues pursued by our paper. Roeger et al. [2009] use a DGSE model with endogenous growth to explore the effect of a number of policies on knowledge investment in the European Union. A key finding is that knowledge investment and productivity increase with a reduction in financial frictions. Bandyopadhyay et al. [2019] explore the effects of heterogeneity in the endowment of human capital in children on productivity, and find that the absence of financial markets to buffer against human capital shocks, total factor productivity is negatively affected.

With respect to permanent consequences of a transitory financial disruption, the literature is small, and it is basically driven by the events of the Great Recession. There are some studies that suggested a break in GDP trend since 2008. Evidence of the possible break in trend in 2008 was found in the literature, in a structural general equilibrium model by Guerron-Quintana and Jinnai [2019], who emphasized the R&D channel. And it also have been found in the empirical work of Huang and Luo [2018]. The notion of a break in trend however has been challenged by Fernald et al. [2017] who argue, using Okun's Law, that output per capita has started to decrease prior to 2008.

On a more technical note, our approach to modelling human capital and heterogeneity shares some elements used in previous literature. We model human capital accumulation similarly to Krebs [2003] and Gottardi et al. [2015], but heterogeneity is introduced assuming idiosyncratic shocks to the marginal utility of consumption as in Lucas [1980], Lucas [1992] and Wen [2015].

This type of heterogeneity is useful because by using this assumption we are able to solve for the individual's policy functions in closed form and which are linear in the relevant individual state and aggregation is simple. Also relevant is that such a construction is amenable to be used to introduce a credit crunch of the type analyzed in for example, Guerrieri and Lorenzoni [2017].

The rest of the document is organized as follows. Section 2 presents the model. Section 3 presents an analysis of the solution of the model, including a derivation of the long-run implications of the credit friction. Section 4 presents an analysis of the consequences of the transitory financial disruption. Section 4 also explores the plausibility of human capital being adversely affected during the Great Recession, the possible role of government interventions during the financial disruption, and the impact of productivity shocks. Finally, Section 5 offers some concluding remarks.

2 The Model

Overview

Households accumulate both physical and human capital, which they provide to a CRS firm to produce output of the economy. This firm is subject to typical productivity shocks.[4] As both factors of production can be accumulated, and the production function exhibits CRS, diminishing returns are eliminated and perpetual growth arises endogenously.[5]

[4] In spite of the model being composed of heterogenous agents, it is flexible enough to accommodate productivity shocks as in the typical RBC framework. Since the model is used to analyze long term issues as well as short term ones, it is desirable to explore the short run implications of productivity shocks to assess if the model is capable to give sensible predictions of a type of shock well studied in the macroeconomic literature. This is done in subsection 4.3.

[5] The initial paper modelling endogenous growth in this manner is Mankiw et al. [2002]. A more recent paper also using this assumption to study Ramsey taxation is Gottardi et al. [2015].

We introduce also heterogeneity and credit in the following manner. We assume that decisions on investment in both types of capital are made at the beginning of the period and cannot be modified until the next period. After choosing their capital levels and within the same period, agents face a preference shock that can make consumption more or less urgent than planned, this shock being idiosyncratic among individuals. They have access to credit in the form of one-period bonds which they can use to finance consumption. Because the preference shock is idiosyncratic, some agents would like to borrow and others to lend, and credit arises endogenously in equilibrium. While agents can borrow to finance consumption, they are exogenously constrained to not borrow more than a fraction of their current resources.[6]

Households

We assume a measure one of agents that live infinite periods and are indexed by $i \in [0,1]$. Then each aims to maximize:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \theta_{i,t} u(c_{i,t}), \quad u(c_{i,t}) = \frac{c_{i,t}^{1-\sigma}}{1-\sigma} \tag{2.1}$$

Where $c_{i,t}$ is consumption of household i in period t and the period utility function is of the CRRA type with risk aversion parameter σ. $\mathbb{E}_0$ refers to the expectation regarding the time path of the idiosyncratic shock $\theta_{i,t}$ and also the aggregate productivity shock to be specified later.[7]

$\theta_{i,t}$ are independent and identically distributed shocks over time and over individuals. This preference shock will be modeled as shifting the marginal utility of consumption, so that, when the value of the shock is high, the household has a strong desire to increase consumption. This

[6]In this way, in subsection 4.2, we can also analyze the effect of a *transitory* decrease in that fraction, a credit crunch that tightens household's feasibility sets and then trace out the implications for GDP's trend.

[7]$\mathbb{E}_0$ can accommodate easily a recurring stochastic credit or liquidity shock, but we will work for ease of exposition with the assumption that the credit crunch is a completely unexpected event and that once the credit crunch is in place, the credit recovery will be perfectly anticipated.

type of shock can be thought of a "urgency to consume" shock or a "liquidity" shock because households facing a large shock are induced to obtain liquid funds to finance higher consumption. Specifically $\theta_{i,t}$ can take two values:

$$\theta_{i,t} = \begin{cases} \theta_\ell & \text{with probability} \quad \pi_\ell \\ \theta_h & \text{with probability} \quad \pi_h \end{cases}, \quad \theta_h > \theta_\ell, \quad \pi_\ell + \pi_h = 1. \tag{2.2}$$

Individuals receiving θ_ℓ face a low liquidity shock, while individuals receiving θ_h receive a high liquidity shock. Agents maximize utility subject to the following budget constraint:

$$k_{i,t+1} + h_{i,t+1} + q_t b_{i,t+1} + c_{i,t} = r_{k,t} k_{i,t} + r_{h,t} h_{i,t} + b_{i,t} \equiv w_{i,t} \tag{2.3}$$

where $k_{i,t}$ and $h_{i,t}$ are physical and human capital respectively. Returns to both types of capital are defined as:

$$r_{k,t} = 1 - \delta + \upsilon_t(1 - \tau_t), \quad r_{h,t} = 1 - \delta + \omega_t(1 - \tau_t) \tag{2.4}$$

υ_t is the rental rate of physical capital and ω_t is the wage rate. δ is the depreciation rate of both types of capital assumed for simplicity to be the same. We also assume that government levies taxes both on labor and capital income, assuming again for simplicity the same tax rate τ_t for both types of income. And $b_{i,t}$ are one period bonds, which are traded at price q_t, the inverse of which determines the real interest rate. In (2.3) we have defined $w_{i,t}$ as total resources available to individual i at the beginning of the period which is composed of all factor income plus any bonds purchased the previous period.[8]

[8] We are using the convention then that a positive $b_{i,t}$ is savings for the household, while a negative $b_{i,t}$ would be borrowing. Also, we are assuming that government can issue bonds, and that both private and government bonds are perfect substitutes.

With resources $w_{i,t}$ at hand, households choose physical and human capital to be rented next period, $k_{i,t+1}$ and $h_{i,t+1}$, and bonds $b_{i,t+1}$ at price q_t. They face also a financial constraint whereby they cannot borrow more than a fraction of their current resources:

$$b_{i,t+1} \geq -\phi w_{i,t}, \quad \phi \geq 0 \tag{2.5}$$

ϕ is an exogenous fraction, and it gives a measure of the degree of the financial friction. This is an important parameter as in this paper will be equated to the depth of financial markets.[9]

Firms

Firms hire labor to produce the output of the economy according to the simple technology:

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha}, \;\; 0 < \alpha < 1 \tag{2.6}$$

where k_t is the aggregate capital stock and h_t is the aggregate human capital stock. A_t is the productivity shock following the process:

$$\ln A_t = (1-\rho)\ln A + \rho \ln A_{t-1} + \epsilon_t \tag{2.7}$$

ρ measures the persistence of the productivity shock, ϵ_t is its innovation and A is the long run value of the productivity shock. In subsection 4.3, we present a discussion of the impulse response functions for this shock.

Timing

We introduce the following timing and information assumptions for households. Facing prices

[9]When analyzing the credit crunch in section 4.2, we will assume a time varying parameter ϕ_t, which will fall at the moment of the crunch, and afterwards recover in a predictable fashion.

and government policy, in the first subperiod, they decide the amounts of physical and human capital stock to be ready to use next period $k_{i,t+1}$ and $h_{i,t+1}$ respectively. Once this decision is made, and within the same period but in a second subperiod, they find out the value of the preference shock $\theta_{i,t}$. The capital investment decisions cannot be modified at this point, but they are able to use credit markets (reflected in bonds $b_{i,t+1}$) to either finance high consumption or to use savings to defer consumption for the future. In this way credit has a role in providing liquidity.[10] Figure 1 shows the timing of information and decisions within a period, as just discussed.

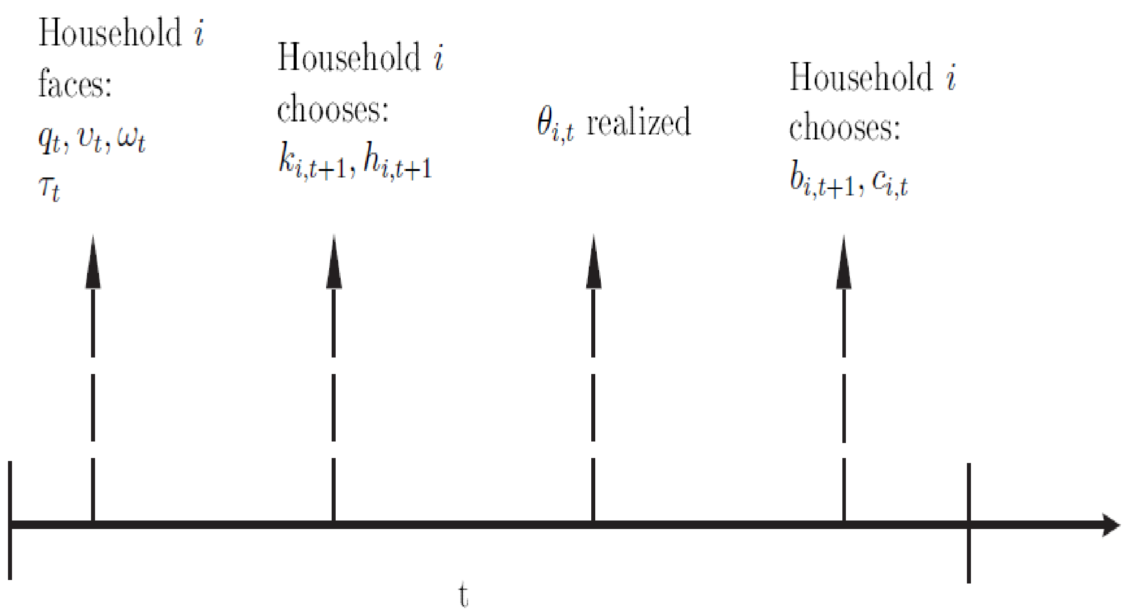


Figure 1: Timing. Not all information is revealed simultaneously. At the beginning of the period, households choose both types of capital, without knowing the actual value of the preference shock. Later on, the preference shock is realized, and then households can use bonds to adjust current consumption or to save for next period.

Government

[10]Modeling both physical and human capital as being illiquid in the sense explained above allow us to use a Guess-and-Verify strategy to solve the model and at the same time allow for analysing the effect of a credit crunch, as will be explained in section 3.

Government issues one period government bonds and uses income tax to finance government expenditure:

$$d_t + g_t = \tau_t \upsilon_t k_t + \tau_t \omega_t h_t + q_t d_{t+1} \tag{2.8}$$

where d_t is government bonds and g_t is unproductive government expenditure. In the next section we analyze different assumptions regarding the exogeneity of either government bonds or government expenditure when analyzing different values of ϕ. Later on when we introduce a credit crunch in subsection 4.2.2, we will specify the specific government policies that the government could use to cope with the financial crises.

We next present the definition of equilibrium.

Definition of Equilibrium

A competitive equilibrium is a sequence of price of bonds, wage rate, rental rate of capital $\{q_t, \omega_t, \upsilon_t\}_{t=0}^{\infty}$*, a sequence of taxes, government expenditure and government bonds* $\{\tau_t, g_t, d_t\}_{t=0}^{\infty}$*, such that:*

1. $h_{i,t+1}, k_{i,t+1}, c_{i,t}$ *and* $b_{i,t+1}$ *maximize individual's utility subject to* (2.3) *and* (2.5).

2. *The bonds market clears:*[11]

$$b_t = d_t \tag{2.9}$$

3. *Demand of both types of capital equal the supplies*

4. *Government satisfies its budget constraint* (2.8)

[11] In general, a variable that corresponds to a household, but without subindex means the aggregate amount over the cross section. In this case b_t is aggregate bonds, the sum of $b_{i,t}$ over i.

3 Analysis

The households' problem

Before solving households' problem, let us define total capital as $z_{i,t} = k_{i,t} + h_{i,t}$ and $\nu_{i,t}$ as the share of physical capital in total capital. We guess and later verify that this share is the same across households, therefore:

$$\nu_t = \frac{k_{i,t}}{k_{i,t} + h_{i,t}} \tag{3.1}$$

The budget constraint (2.3) can then be written as:

$$z_{i,t+1} + q_t b_{i,t+1} + c_{i,t} = \left[r_{k,t}\nu_t + r_{h,t}(1-\nu_t)\right] z_{i,t} + b_{i,t} \equiv w_{i,t} \tag{3.2}$$

This version of the budget constraint will be easier to work with than the original. Note that $z_{i,t}$ and ν_t are both endogenous variables.

Let total resources at hand at the beginning of the period $w_{i,t}$ be the state variable for households.[12] The Bellman equation is:

$$\mathcal{V}_t(w_{i,t}) = \max_{z_{i,t+1},\nu_{t+1}} \left\{ \sum_{j=\ell,h} \max_{c_{i,t},b_{i,t+1}} \left[\theta_{j,t} u(c_{i,t}) + \beta \mathbb{E}_t \mathcal{V}_{t+1}(w_{i,t+1})\right] \pi_j \right\} \tag{3.3}$$

subject to (3.2) and (2.5). Note how the timing of the model is reflected in the optimization problem that households face in (3.3). They choose $z_{i,t+1}$ and ν_{t+1} acting optimally, considering in the inner problem in (3.3) -the problem in braces-, the expected value for welfare induced by the preference shock to hit later in the period.

[12]Other state variables are all prices of the economy and variables such as government expenditures, government bonds and taxes, they are all subsumed in the subindex t in the value function of the Bellman equation.

To solve the model we use a guess and verify approach. Let us use the following guess for the value function:

$$\mathcal{V}_t(w_{i,t}) = \psi_t u(w_{i,t}) = \psi_t \frac{w_{i,t}^{1-\sigma}}{1-\sigma} \tag{3.4}$$

where ψ_t is a yet undetermined stochastic coefficient to be determined later on.

The Appendix A.1 details the procedure for the method of solution employed. Here we discuss the general features of the solution. Because of the simple structure assumed in (2.2), it is possible to derive closed form solutions for the relevant policy functions for households. Let us guess then that at the beginning of each time period, a fraction μ_t is accumulated as capital:

$$z_{i,t+1} = \mu_t w_{i,t} \tag{3.5}$$

Note that μ_t does not depend on individuals' specific value of $\theta_{i,t}$ since at that moment of the current period such value is unknown and it is serially uncorrelated. Then households in the second subperiod face net wealth given by $(1-\mu_t)w_{i,t}$. As can be seen in (3.2), net wealth can be used for consumption or savings.

Conditional on a given value of μ_t, is possible to find the policy functions for households facing a low and a high preference shock, respectively. It is a natural conjecture, given the two-shock situation expressed in (2.2) that household i will be a borrower, when $\theta_i = \theta_h$ and it will be a lender when $\theta_i = \theta_\ell$. Hence those households facing a low shock will not bind their financial constraint (2.5) and their policies will be:[13]

$$c_{i,t}^{\ell} = (1-\zeta_t)(1-\mu_t)w_{i,t}, \quad q_t b_{i,t}^{\ell} = \zeta_t(1-\mu_t)w_{i,t} \tag{3.6}$$

[13]The supraindex ℓ used in equations (3.6) denote the values of these variables for an individual i facing a low shock.

where ζ_t is the savings rate out of net wealth. Appendix A.1 shows that optimally, this value will satisfy:

$$\theta_\ell \left[(1-\zeta_t)(1-\mu_t)\right]^{-\sigma} q_t = \beta \mathbb{E}_t \psi_{t+1} \left\{ \left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right]\mu_t + \frac{\zeta_t}{q_t}(1-\mu_t) \right\}^{-\sigma} \tag{3.7}$$

which states that the marginal cost of carrying one unit of consumption to next period must equal the expected marginal benefit. The marginal benefit, which involves the slope of the value function, in this case has an analytical expression due to assumption (3.4).

Households facing a high preference shock on the other hand, will bind the financial constraint (2.5) and then their policy functions are given by:

$$c^h_{i,t} = (q_t\phi + 1 - \mu_t)w_{i,t}, \quad b^h_{i,t+1} = -\phi w_{i,t} \tag{3.8}$$

The policy functions found have the virtue of being linear in the state $w_{i,t}$ and hence aggregation is straightforward, this will be accomplished in the next subsection. Households need to decide at the beginning of the period how much of both physical and human capital to choose for next period (to choose μ_t and ν_{t+1}). In order to do so, they will take into account the idiosyncratic uncertainty of the preference shock, and the likelihood of ending up using policies (3.6) or (3.8).

The Appendix shows that the following equations need to be satisfied for such optimal behavior:

$$\begin{aligned}
- \quad & \pi_\ell \theta_\ell \left[(1-\zeta_t)(1-\mu_t)\right]^{-\sigma} \left[\frac{d\zeta_t}{d\mu_t}(1-\mu_t) + (1-\zeta_t)\right] \\
+ \quad & \pi_\ell \beta \mathbb{E}_t \psi_{t+1} \left\{\left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right]\mu_t + \frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{-\sigma} \\
& \left\{r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1}) + \frac{1}{q_t}\left[\frac{d\zeta_t}{d\mu_t}(1-\mu_t) - \zeta_t\right]\right\} - \pi_h \theta_h (q_t\phi + 1 - \mu_t)^{-\sigma} \\
+ \quad & \pi_h \beta \mathbb{E}_t \psi_{t+1} \left\{\left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right]\mu_t - \phi\right\}^{-\sigma} \\
& \left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right] = 0
\end{aligned} \tag{3.9}$$

and:

$$\begin{aligned}
- \quad & \pi_\ell \theta_\ell \left[(1-\zeta_t)(1-\mu_t)\right]^{-\sigma} \left[\frac{d\zeta_t}{d\nu_{t+1}}(1-\mu_t)\right] \\
+ \quad & \pi_\ell \beta \mathbb{E}_t \psi_{t+1} \left\{\left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right]\mu_t + \frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{-\sigma} \\
& \left\{(r_{k,t+1} - r_{h,t+1})\nu_{t+1} + \frac{1}{q_t}\left[\frac{d\zeta_t}{d\nu_{t+1}}(1-\mu_t)\right]\right\} \\
+ \quad & \pi_h \beta \mathbb{E}_t \psi_{t+1} \left\{\left[r_{k,t+1}\nu_{t+1} + r_{h,t}(1-\nu_{t+1})\right]\mu_t - \phi\right\}^{-\sigma} (r_{k,t+1} - r_{h,t+1})\mu_t = 0
\end{aligned} \tag{3.10}$$

(3.9) and (3.10) being the FOC for μ_t and ν_{t+1} respectively, in the optimization problem (3.3). The expressions $d\zeta_t/d\mu_t$ and $d\zeta_t/d\nu_{t+1}$ are the derivatives of the savings rate ζ_t with respect to μ_t and ν_{t+1} respectively and are explicitly derived in the Appendix A.1, equations (A.1l) and (A.1m). Note that in using (3.1) we assumed that the fraction of physical capital in total capital was the same across households, equation (3.10) confirms that such an assumption holds in equilibrium as no variable in that equation is indexed by i.

Appendix A.1 also shows that as part of the guess and verify method, the equation for ψ_t of

(3.4) satisfies the recursive equation:

$$\begin{aligned}\psi_t &= \pi_\ell\theta_\ell[(1-\zeta_t)(1-\mu_t)]^{1-\sigma} + \pi_\ell\beta\mathbb{E}_t\psi_{t+1}\left\{[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})]\,\mu_t + \frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{1-\sigma} \\ &+ \pi_h\theta_h(q_t\phi + 1 - \mu_t)^{1-\sigma} + \pi_h\beta\mathbb{E}_t\psi_{t+1}\left\{[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})]\,\mu_t - \phi\right\}^{1-\sigma} \quad (3.11)\end{aligned}$$

The firms' problem

The firms' problem is standard and simple. Maximizing profits, the firm equates marginal products of both their inputs with rental prices:

$$\upsilon_t = A_t\alpha\left(\frac{h_t}{k_t}\right)^{1-\alpha} = A_t\alpha\left(\frac{1-\nu_t}{\nu_t}\right)^{1-\alpha}, \quad \omega_t = A_t(1-\alpha)\left(\frac{k_t}{h_t}\right)^{\alpha} = A_t(1-\alpha)\left(\frac{\nu_t}{1-\nu_t}\right)^{\alpha} \quad (3.12)$$

where it has been used the definition of ν_t in (3.1).

3.1 Aggregate relationships

Aggregation is easy in the model due to the assumptions employed regarding the utility function and the absence of serial correlation for the preference shocks. Aggregating the policy functions (3.6) and (3.8) derived above:[14]

$$q_t b_{\ell,t+1} = \zeta_t(1-\mu_t)\pi_\ell w_t, \; c_{\ell,t} = (1-\zeta_t)(1-\mu_t)\pi_\ell w_t, \; b_{h,t+1} = -\phi\pi_h w_t, \; c_{h,t} = (q_t\phi + 1 - \mu_t)\pi_h w_t \quad (3.13)$$

[14]The subindex ℓ denotes the aggregate value of the variable for those individuals facing a low shock in a given time period, for example $b_{\ell,t+1}$ is the aggregate amount of bonds demanded by low shock individuals. A similar notation is used for high shock individuals.

where w_t is aggregate wealth which equals, from (3.2) and imposing market clearing in the bonds market:[15]

$$w_t = [r_{k,t}\nu_t + r_{h,t}(1-\nu_t)]z_t + b_t \tag{3.14}$$

where z_t is the beginning of period aggregate total capital stock and b_t is the beginning of period aggregate bonds.

Recall from (3.5) that all individuals accumulate capital by choosing a fraction μ_t of beginning of period total resources, therefore we can obtain the aggregates:

$$z_{\ell,t+1} = \mu_t \pi_\ell w_t, \quad z_{h,t+1} = \mu_t \pi_h w_t \tag{3.15}$$

Variables can further be aggregated by summing amounts for both groups. Total consumption, bonds and capital are given from (3.13) and (3.15):

$$c_t = [(1-\zeta_t)(1-\mu_t)\pi_\ell + (q_t\phi + 1 - \mu_t)\pi_h]w_t,\ b_{t+1} = \left[\frac{\zeta_t}{q_t}(1-\mu_t)\pi_\ell - \phi\pi_h\right]w_t,\ z_{t+1} = \mu_t w_t \tag{3.16}$$

Several macroeconomic relationships can be analyzed in the model with the solutions obtained so far. For example Appendix A.2 show how the model satisfies the national income identity that total output must equal aggregate consumption plus investment in both types of capital plus government expenditures. Although the economy is well defined in the model, perpetual growth implies that in order to solve the model we need a normalization, which is tackled next.

Normalization

[15] Note that we imposed the fact that under market clearing $d_t = b_t$ and with some abuse of notation b_t will also denote the *equilibrium* value of bonds.

Expressions in (3.16) are simple enough to carry on with the solution of the model. The economy perpetually growing however means that these expressions are not stationary. We choose to normalize the economy by the initial period aggregate capital stock z_t. Let hats over variables denote the variable divided by the capital stock, for example $\hat{c}_t = c_t/z_t$. Then expressions in (3.16) are written as:

$$\hat{c}_t = [(1-\zeta_t)(1-\mu_t)\pi_\ell + (q_t\phi + 1 - \mu_t)\pi_h]\hat{w}_t,\ \Gamma_{z,t+1}\hat{b}_{t+1} = \left[\frac{\zeta_t}{q_t}(1-\mu_t)\pi_\ell - \phi\pi_h\right]\hat{w}_t,\ \Gamma_{z,t+1} = \mu_t\hat{w}_t \tag{3.17}$$

Where $\Gamma_{z,t+1} = z_{t+1}/z_t$ is the rate of growth of total capital and:

$$\hat{w}_t = r_{k,t}\nu_t + r_{h,t}(1-\nu_t) + \hat{b}_t \tag{3.18}$$

using (3.14). Useful expressions can also be obtained by simple manipulations of the definitions of both $\Gamma_{z,t}$ and ν_t:[16]

$$\Gamma_{z,t+1} = \Gamma_{k,t+1}\nu_t + \Gamma_{h,t+1}(1-\nu_t),\ \ \nu_{t+1} = \frac{\Gamma_{k,t+1}}{\Gamma_{z,t+1}}\nu_t \tag{3.20}$$

where $\Gamma_{k,t+1} = k_{t+1}/k_t$ and $\Gamma_{h,t+1} = h_{t+1}/h_t$, are the rates of growth of physical and human capital respectively.

The production function in (2.6) can also be written in terms of rates of growth:

$$\Gamma_{y,t} = \frac{A_t}{A_{t-1}}\Gamma_{k,t}^{\alpha}\Gamma_{h,t}^{1-\alpha} \tag{3.21}$$

[16]For example $\Gamma_{z,t+1} = z_{t+1}/z_t$ is equal to:

$$\frac{z_{t+1}}{z_t} = \frac{k_{i,t+1} + h_{i,t+1}}{k_{i,t} + h_{i,t}} = \frac{k_{i,t+1}}{k_{i,t} + h_{i,t}} + \frac{h_{i,t+1}}{k_{i,t} + h_{i,t}} = \frac{k_{i,t+1}/k_{i,t}}{(k_{i,t} + h_{i,t})/k_{i,t}} + \frac{h_{i,t+1}/h_{i,t}}{(k_{i,t} + h_{i,t})/h_{i,t}} \tag{3.19}$$

Using the definition of ν_t in this equation, we obtain the expression. The expression for ν_{t+1} can likewise be obtained with simple substitutions.

Where $\Gamma_{y,t} = y_t/y_{t-1}$. We close the model with the government budget constraint, equation (2.8), which upon normalization can be written as:

$$\hat{b}_t + \hat{g}_t = \tau_t[\upsilon_t \nu_t + \omega_t(1-\nu_t)] + q_t \hat{b}_{t+1}\Gamma_{z,t+1} \tag{3.22}$$

System of equations and variables of interest

The dynamic system to be solved consists of 14 equations. The optimality condition for next period total capital, equation (3.9). How total capital is divided among physical and human capital, equation (3.10). The optimality of the savings rate, equation (3.7). The stochastic recursive equation for the unknown parameter ψ_t, equation (3.11). The optimality conditions for both rental rates in equations (3.12). The normalized equations for consumption, next period bonds and the rate of growth of capital, equations in (3.17). The equation for normalized wealth (3.18). The behavior of growth of total capital and behavior of the fraction of physical capital to total capital, equations in (3.20). The growth of output, equation (3.21) and finally, the normalized government budget constraint, equation (3.22). These equations form a system to be solved for the unknowns $\mu_t, \nu_t, \zeta_t, \psi_t, \upsilon_t, \omega_t, \hat{c}_t, \hat{b}_t, \Gamma_{z,t}, \hat{w}_t, \Gamma_{k,t}, \Gamma_{h,t}, \Gamma_{y,t}$ and q_t.

Several other variables of interest can of course be constructed from the equations above. For example, we can obtain several typical macro variables of interest such as investment over GDP, consumption over GDP, government debt over GDP and government expenditure over GDP. For example to find gross investment over GDP one could compute:

$$\frac{x_t}{y_t} = \frac{k_{t+1} - (1-\delta)k_t}{A_t k_t^{\alpha} h_t^{1-\alpha}} = \frac{\Gamma_{k,t+1} - 1 + \delta}{A_t (h_t/k_t)^{1-\alpha}} = \frac{\Gamma_{k,t+1} - 1 + \delta}{A_t}\left(\frac{\nu_t}{1-\nu_t}\right)^{1-\alpha} \tag{3.23}$$

To find consumption over GDP we could use:

$$\frac{c_t}{y_t} = \frac{z_t}{y_t}\hat{c}_t = \frac{k_t + h_t}{A_t k_t^{\alpha} h_t^{1-\alpha}}\hat{c}_t = \frac{(k_t + h_t)/k_t}{A_t (h_t/k_t)^{1-\alpha}}\hat{c}_t = \frac{\hat{c}_t}{\nu_t A_t}\left(\frac{\nu_t}{1-\nu_t}\right)^{1-\alpha} = \frac{\hat{c}_t}{A_t \nu_t^{\alpha}(1-\nu_t)^{1-\alpha}} \tag{3.24}$$

To find government debt over GDP and government expenditure over GDP we could use similar manipulations to obtain:

$$\frac{g_t}{y_t} = \frac{\hat{g}_t}{A_t \nu_t^{\alpha}(1-\nu_t)^{1-\alpha}}, \ \frac{b_t}{y_t} = \frac{\hat{b}_t}{A_t \nu_t^{\alpha}(1-\nu_t)^{1-\alpha}} \tag{3.25}$$

Another variable of interest would be the total amount of private credit in the economy. Since it is high shock individuals who obtain credit by selling bonds, we could use $b_{h,t+1}$ in (3.13) to first find credit over total capital: $\hat{b}_{h,t+1} = -\phi \pi_h \hat{w}_t / \Gamma_{z,t+1}$. Then with similar manipulations as in (3.24) and (3.25) we have that private credit over GDP is:

$$\frac{b_{h,t+1}}{y_t} = \frac{\hat{b}_{h,t+1}}{A_t \nu_t^{\alpha}(1-\nu_t)^{1-\alpha}} \tag{3.26}$$

The variables derived will serve for both the long run analysis and the short run analysis of future sections. Now we turn to the long run analysis. We view ϕ as measuring the degree of financial depth, and we solve for the steady state of the model, for different values of ϕ and derive testable implications of the model.

3.2 Long run implications of the credit friction: implications of financial depth

The steady state of the model, a situation where the productivity innovation is muted and productivity is set equal to its average value of A is the natural setup to investigate the effect of "financial deepening" measured by ϕ on growth. Note that households are continuously being buffeted by the idiosyncratic preference shock $\theta_{i,t}$. So we have a situation where credit exists in equilibrium in the steady state. Due to the simplifying assumptions made in the model, it is straightforward to analyze the steady state, again there is no need to compute distributions, which facilitates greatly the analysis.

Appendix A.3 shows that in the steady state, both returns to physical capital and human capital must be the same, as both types of capital are chosen at the beginning of the period, $r_k = r_h \equiv r$. It is also shown that it must be the case that $\upsilon = \omega$ and $\nu = \alpha$ and that the following equality is satisfied:

$$\Gamma \equiv \Gamma_h = \Gamma_k = \Gamma_z = \Gamma_y \tag{3.27}$$

Where Γ is defined as the rate of growth of the economy.[17] We choose to compute several standard variables in the macro literature. First focus on the investment and consumption share of GDP, from (3.23) and (3.24):[18]

$$\frac{x_t}{y_t} = \frac{\Gamma - 1 + \delta}{A}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}, \quad \frac{c_t}{y_t} = \frac{\hat{c}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{3.28}$$

The expenditure and government debt over GDP, from (3.25):

$$\frac{g_t}{y_t} = \frac{\hat{g}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}}, \quad \frac{b_t}{y_t} = \frac{\hat{b}}{A\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \tag{3.29}$$

[17]The last equality in (3.27) follows immediately from the definition of the rate of growth of GDP, in steady state, equation (3.21).

[18]In what follows, variables without a subindex t, such as Γ and $\hat{c}$ corresponds to the variable in steady state, which are of course time invariant.

and the fraction of private credit over GDP, from (3.26):

$$\frac{b_{h,t+1}}{y_t} = -\frac{\phi \pi_h \hat{w}}{\Gamma A \nu^{\alpha}(1-\nu)^{1-\alpha}} \tag{3.30}$$

Our objective is to solve the model in the steady state for different values of the parameter ϕ. In order to do so, we need to solve the resulting non-linear system of equations several times; this will be accomplished numerically, as there is no closed form solution. Two conceptual considerations drive this exercise. First we want to analyze what testable implications we can derive with the model. This we believe is important, because as discussed in the introduction we are not aware of a theoretical contribution in the growth literature that takes jointly into consideration human capital accumulation, growth and credit frictions. For this exercise we will focus on testable implications for the rate of growth of GDP, investment over GDP, consumption over GDP, private credit over GDP, and the price of bonds, which gives us a measure of the interest rate. Second, as previously noted, the empirical literature has found a non-monotonic relationship between financial depth and long run growth. We want then to assess if the model can deliver such prediction.

The aim of the model is not to give exact quantitative predictions of the effect of the financial friction on the economy. Notwithstanding, even qualitative predictions will likely depend on the specific parametrization of the model. The next section will develop a rough calibration of the model in order to analyze the consequences of a credit crunch. We use that calibration as a point of departure in this section. According to the calibration of the next section, the value for the credit friction parameter is $\phi = 0.1458$. We compute the steady state of the model for values above and below this parameter. Different values of ϕ will induce different policy functions and different equilibria of the model, but in order to close the model we need to take a stance on government behavior under different credit conditions. In the steady state, equation (A.3e) in

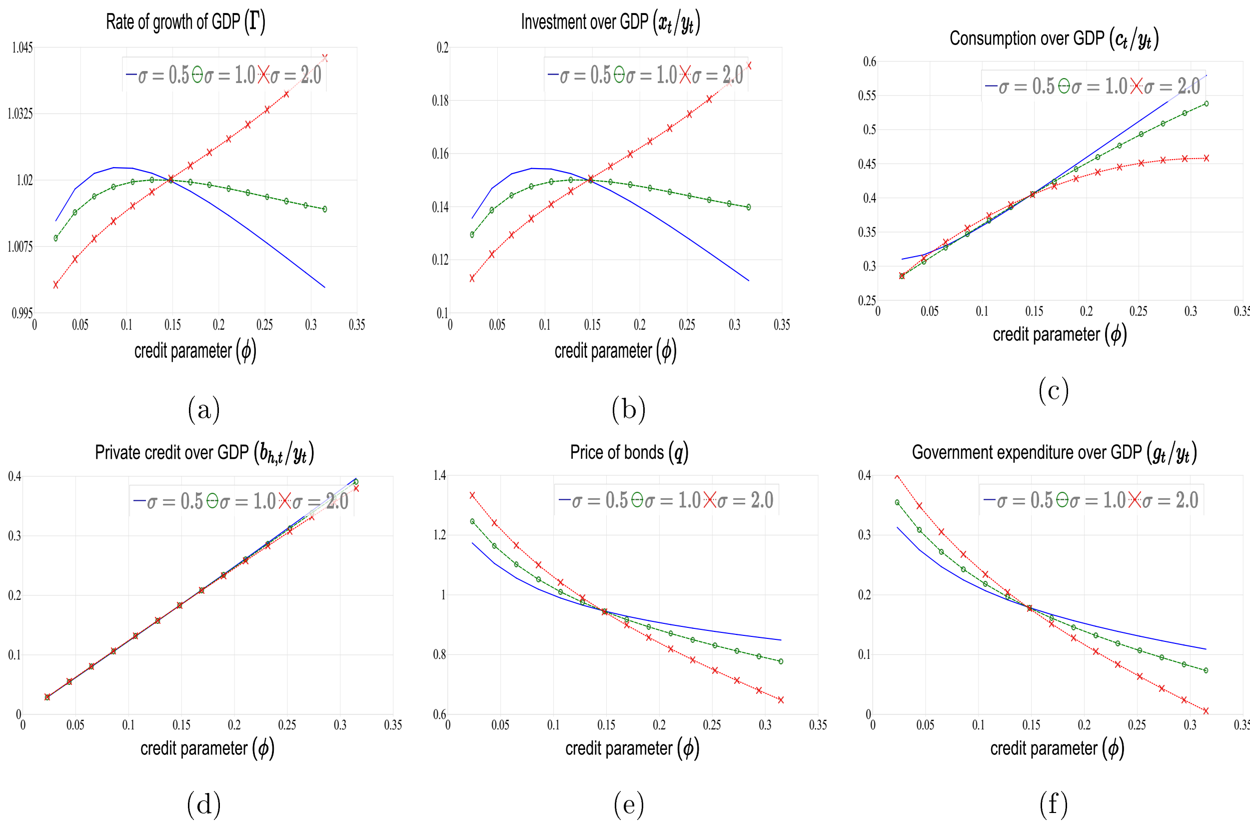


Figure 2: Some variables as function of ϕ when $\hat{g}$ adjusts to satisfy the government budget constraint.

Appendix A.3 gives the government budget constraint, which we repeat here for convenience:

$$\hat{b} + \hat{g} = \tau\omega + q\hat{b}\Gamma \tag{3.31}$$

Because the government budget constraint must always be satisfied, different ϕ will give different values for q and Γ. We choose to analyze two cases such that this constraint is satisfied, one when government adjusts $\hat{g}$ and the other when it adjusts $\hat{b}$.[19] Figure 2 shows the case when normalized government expenditure adjusts to satisfy the government budget constraint. We chose to do a sensitivity analysis of the risk aversion parameter σ for each of the variables

[19] In principle, the tax rate τ could also be modified to accomplish the exercise, but we think it is more realistic to change either the "size of the government" or the level of debt, as changes in taxation are historically less common and their long run behavior is tied to legislation.

portrayed in the figure.

We can see in Figures 2a, 2b and 2c that the behavior of the rate of growth, investment over GDP and consumption over GDP may be non-monotone depending on the value of σ. Starting from low values of ϕ, as it increases, both investment and consumption over GDP increase. For relatively low values of σ however (specifically for values less or equal than one), investment over GDP start to decrease along with the economy's growth rate, while consumption over GDP still increases strongly. Some intuition for this result can be obtained in spite of the model's complexity by recalling the timing of decisions and the markets involved. The reason why households wouldn't invest large amounts of capital (both physical and human) is because of the risk of facing a high urgency to consume later in the period. The possibility of credit mitigates this risk as they can resort to the credit market without sacrificing investments. Moreover, higher investments would allow households to more easily repay (in later periods) any credit undertaken within the current period. When ϕ increases there is an incentive to further increase capital investments, shown in the figure as ϕ increases. For higher values of ϕ however, the interest is also high which can be observed in Figure 2e. This means that households facing a low preference shock have a positive income effect for their savings in bonds, which may become a disincentive for savings in capital and for growth. For relatively low values of σ the income effect of a high interest rate maybe dominant, and this could explain that growth decreases as compared to the case when σ is high, for example the case $\sigma = 2$ in the figure.

As Figures 2d, 2e and 2f show, private credit over GDP, the price of bonds and government expenditure over GDP are always monotone functions of ϕ, independently of the value of σ. As the credit parameter increases, the supply of bonds increases. High shock individuals are able to get more resources from low shock individuals, and the price of bonds decreases, increasing the interest rate. Savings from low shock individuals are not used to finance government as

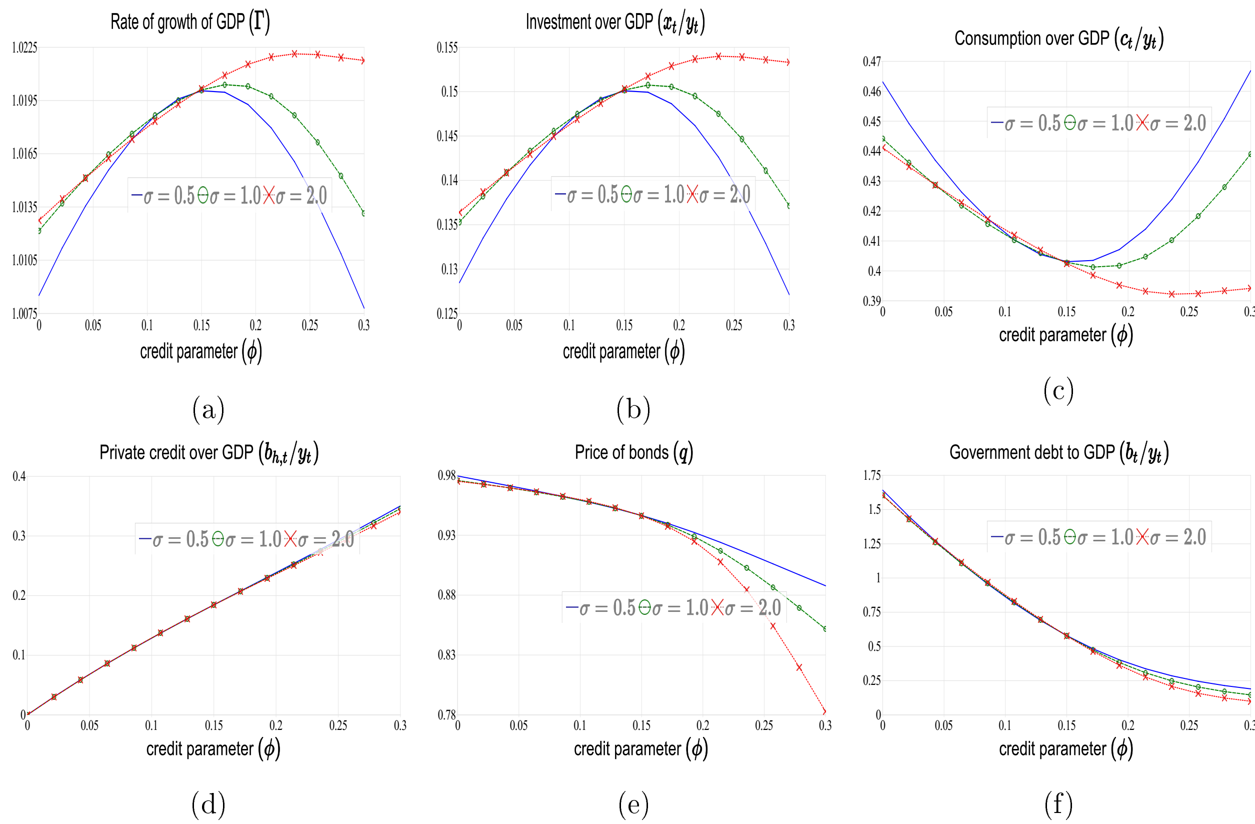


Figure 3: Some variables as function of ϕ when $\hat{b}$ adjusts to satisfy the government budget constraint.

its debt is fixed in normalized terms. Looking at equation (3.31), because $\tau\omega$ is fixed, the behavior of normalized government expenditure is tied to the behavior of the price of bonds and the rate of growth of the economy. Even when there is a positive effect on growth when ϕ increases, because the interest rate is positively affected, the term $q\Gamma$ in (3.31) decreases and the government finds it more expensive to service its debt. It is therefore forced to cut down on government expenditure.

We now turn to the case where government debt adjusts to satisfy the government constraint while normalized government expenditure remains fixed; Figure 3 presents this case. In the bottom of Figure 3, figures 3d, 3e and 3f show that private credit over GDP, the price of bonds and normalized government debt are monotone functions of ϕ and their behavior is similar to the case described in Figure 2. Private credit over GDP increases with ϕ as expected. The interest rate, as in the previous case, increases with ϕ, as supply of bonds increase and in equilibrium q must fall. The price of bonds times the rate of growth $q\Gamma$ decreases with ϕ, then the cost of serving its obligations increases with ϕ for government and then it is forced to cut down on debt, this is shown in Figure 3f.

Regarding the rate of growth of the economy and investment and consumption over GDP, the general pattern in figures 3a, 3b and 3c are roughly similar independently of the value of σ. For low values of ϕ, increases of this parameter produces increases in growth and investment over GDP and decreases in consumption over GDP. Beyond a certain point, however, further increases in ϕ are detrimental for growth and for investment over GDP, and lead to increases in consumption over GDP. Because the point where the curves intersect corresponds to the calibrated value of credit frictions determined in subsection 4.1, Figure 3a implies that the US would initially experience positive growth if there is more financial deepening when $\sigma = 2$, arguably the empirically relevant case. However, these gains will come at decreasing rates and growth will be negatively affected when ϕ reaches roughly 25%.

The intuition behind the detrimental effect on growth for high values of ϕ and low values of σ is similar to the previous case. Figure 3e shows that the interest rate increases when ϕ increase independent of the value of σ. The income effect of a higher interest rate for high values of ϕ incentivizes lenders to use bonds to finance future consumption instead of accumulating both types of capital, in detriment of growth. Note also that since normalized government bonds decrease, the higher savings from low shock individuals are used to finance consumption by high shock individuals who increase their bond selling. This positive effect on consumption is displayed in Figure 3c, in which consumption over GDP increases for high values of ϕ.

Summary of the testable implications of the model

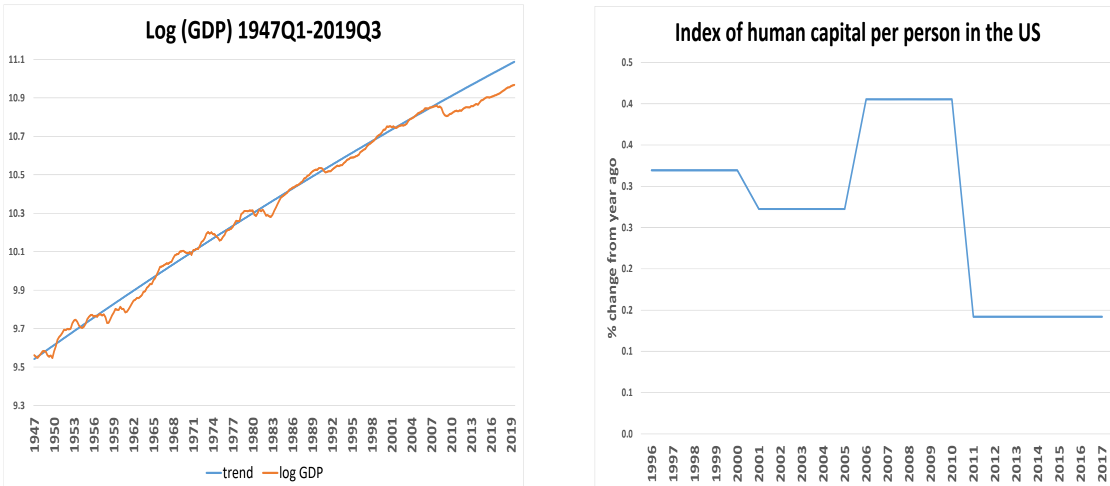
As we noted in the introduction, there are few theoretical general equilibrium models that mix human capital, growth and financial deepening. The model described above gives some predictions regarding the correlations of several macro variables with financial conditions that we think are of value.

Regarding long run growth, and depending on the values of the parameters and assumptions regarding government behavior, we find either a positive relationship or a concave relationship with financial deepening. In some cases, we find an inverted U shaped relationship, something that has been observed in the empirical literature, for example in Arcand et al. [2015] and Law and Singh [2014]. With respect to investment over GDP, we find either a positive relationship or an inverted U shape relationship. For the case of consumption over GDP, when government adjusts government expenditure we find a positive relationship, in some cases with diminishing rates. But when the adjustment is in government debt, we find that consumption over GDP presents a U shaped relationship. Finally, it does not matter if the adjustment is through government expenditure or government debt, there is always a negative relationship between financial deepening and the price of bonds. This is true independently of the value of σ. Therefore, higher financial depth is related to higher interest rates.[20]

4 Dynamics

In this section we are interested in the model's implications regarding the dynamics. We mainly explore the model's predictions for implications of a transitory credit crunch, discussed in subsection 4.2. For completeness we also explore the effects of productivity shocks, as this type of

[20]It is clear also from Figures 2e and 3e that when the adjustment is through government expenditure, the relationship between financial deepening and the price of bonds is convex, whereas it is concave when the adjustment is through government debt.



(a) Log of GDP 1947Q1-2019Q3. Quadratic trend 1947Q1-2008Q4, extrapolated.

(b) Percent change in US human capital, 1966 – 2017.

Figure 4: GDP trend and human capital

shock is commonly used in the macro literature and the model is sufficiently general to make predictions for the macroeconomy when the economy is hit with these type of shocks, as shown in subsection 4.3.

As for the credit crunch, we are particularly interested in fluctuations in the parameter ϕ, the maximum fraction of household resources that can be borrowed. The Great Recession started with a sizable but temporary disruption in the credit markets, and the economy's trend seemed to be affected in a persistent way. Figure 4a shows the logarithm of GDP from the first quarter of 1947 to the third quarter of 2019. We also computed a quadratic trend from the first quarter of 1947 to the last quarter of 2008, and it is extrapolated to the third quarter of 2019. It seems that the level of GDP has been persistently affected, even though the credit crunch was itself transitory.[21] Since human capital is the engine of growth in the model, any negative effect of

[21] The possible break in trend in 2008 is of course not a settled issue, as decomposing a series to extract out a cycle and a trend is always a tricky business. For example, as mentioned in the introduction, Fernald et al.

the credit crunch on economic activity would have to come through a decrease in human capital accumulation. We do not have a comprehensive measure of human capital (and are unaware of the availability of such a measure), but Penn World Table data, discussed further in subsection 4.2.1, shows that it is possible that the rate of human capital accumulation decreased following the Great Recession, as shown in Figure 4b. If the human capital channel was indeed affected in reality, would the model deliver a similar pattern for GDP's level, when ϕ decreases *transitorily*? We explore the answer to this question in this section.

Again, this model was not constructed for specific quantitative prescriptions. But even as a qualitative exercise, the implications of a credit crunch may change depending on different parametrizations, and a realistic parametrization is necessary. Hence here we proceed to obtain a rough calibration of the model in steady state having in mind the situation prior to the Great Recession. Most of the targets involved will correspond to the year 2007. Once the calibration is obtained we will assume that departing from this steady state, the economy unexpectedly suffers a credit crunch. The parameter ϕ will now be time varying, it will decrease unexpectedly and recover gradually but fully in a predictable fashion.

4.1 Calibration

We calibrate the model for a yearly economy. We start by assuming $\delta = 0.06$ as annual depreciation for both types of capital. We also assume $\alpha = 0.36$, as is typical in the macro literature, Appendix A.3 shows then that $\nu = 0.36$. We want to target the annual rate of growth of the economy of two percent, $\Gamma = 1.02$. We also want to target a long-run measure of gross investment to GDP which is roughly 0.15 in the data. x_t/y_t from equation (3.28) then gives a value

[2017] argue, using Okun's Law, that a decline in output per capita began prior to 2008.

of A of:

$$A = \frac{\gamma - 1 + \delta}{0.15} \left(\frac{\nu}{1-\nu} \right)^{1-\alpha} = 0.369. \tag{4.1}$$

With this value at hand, expression υ from the steady state version of the first equation in (3.12) can be obtained as $\upsilon = A\alpha((1-\alpha)/\alpha)^{1-\alpha} = 0.192$. Then, as shown in Appendix A.3, returns on both types of capital must be equal: $r \equiv r_k = r_h$. And we also assume $\tau = 0.20$. With these values at hand we can focus on the first equation of (2.4) to get $r = 1 + \delta + (1-\tau)\upsilon = 1.0936$, an annual return on both types of capital of 9%.

Next we want to target government debt as a fraction of GDP as of 2007, a year prior to the crisis, this value is roughly 0.6. In view of the second equation in (3.29), we have $\hat{b} = A(b_t/y_t)\alpha^\alpha(1-\alpha)^{1-\alpha}$. Replacing b_t/y_t with the data counterpart of 0.6, and using the values for A and α already obtained, we get $\hat{b} = 0.1152$.

Focusing now on the third equation in (3.17), the rate of growth of total capital, in steady state being equal to the rate of growth of GDP, must equal $\Gamma = \mu\hat{w}$, where $\hat{w} = r + \hat{b}$.[22] Therefore we can find the fraction of wealth devoted to capital accumulation as: $\mu = \Gamma/(r + \hat{b}) = 0.844$.

We also want to target the share of government expenditures as a fraction of GDP, which as of 2007 equaled roughly 0.18. From the first equation in (3.29) we have: $\hat{g} = A(g_t/y_t)\alpha^\alpha(1-\alpha)^{1-\alpha}$. Replacing g_t/y_t with the data counterpart of 0.18, we obtain $\hat{g} = 0.03456$.

Now we turn to the model's government budget constraint, equation (3.31). All of the values that appear in this equation are already calibrated so far, except for q. Then we find the value of q that satisfies this equation, which is equal to: $q = 0.9536$. This corresponds to an annual

[22]Most of the equations used in this subsection can be found also in Appendix A.3, for example this equation is the same as the third equation in (A.3d).

interest rate of 4.86%. Note that compared to $r = 1.093$, the return on bonds is lower by 4.5%. This is the "liquidity premium" of capital, which in equilibrium yields a higher return to compensate for the fact that it is more illiquid than bonds.

To calibrate π_ℓ and π_h, we use studies that estimate the fraction of credit constrained households which are in the order of 20%.[23] Therefore we set $\pi_\ell = 0.8$ and $\pi_h = 0.2$.

To calibrate ϕ we take into account the amount of total private debt in the economy as of 2007 which was 18% of GDP.[24] Then because high shock individuals are credit constrained in the model, we have from equation (3.30):

$$0.18 = -\frac{b_{h,t+1}}{y_t} = \frac{\phi \pi_h \hat{w}}{\Gamma A \alpha^\alpha (1-\alpha)^{1-\alpha}} \tag{4.2}$$

From which we obtain a value of $\phi = 0.1458$. This is the value at which all curves in Figures 2 and 3 intersect.

To calibrate ζ, we use market clearing in the bonds market in steady state, which equals to the middle equation in (A.3d) :

$$\Gamma \hat{b} = \left[\frac{\zeta}{q}(1-\mu)\pi_\ell - \phi \pi_h \right] \hat{w} \tag{4.3}$$

So the ζ that satisfies market clearing is $\zeta = 0.964$.[25]

[23] Japelli [1990] found that about 12 percent of households are credit constrained, rising to 19 percent if discouraged borrowers are included. Other literature mentioned in this study also pointed to a value close to 20%. Sensitivity analysis of this parameter showed that it matter little for the qualitative results of the paper.

[24] This number comes from the Flow of Funds. Households liquid assets are considered to be the sum of all deposits plus securities held directly by households.

[25] This value is considered unrealistically large. One of the targets that we choose to satisfy is government debt. Because only low shock individuals save and we are working in a closed economy, we are forcing this group to be the ones that supply the necessary funds for both government and high shock individuals. A more realistic modeling of the idiosyncratic shock or considering an open economy would help to bring more realism. However, when exploring different parameterizations of the model, we found that there is little difference for the qualitative results of the paper.

For the risk aversion parameter, we set $\sigma = 2$ in the baseline specification, and check the sensitivity of the results to changes in this parameter later on. We also normalize $\theta_\ell = 1$. The remaining parameters of the model needed to calibrate are β, θ_h and ψ. These are found by solving the nonlinear system composed of the FOC with respect to μ, equation (A.3a); the equation of optimality of the savings rate, equation (A.3b); and the equation determining ψ, equation (A.3c). We obtain the values $\beta = 0.90$, $\theta_h = 6180$ and $\psi = 40320$.[26]

4.2 The effects of the credit crunch

We assume that the economy was in steady state as of 2007 and in 2008 unexpectedly ϕ is no longer constant. It drops to zero (representing an unexpected decrease in credit availability) at impact and recovers relatively fast in a predictable fashion. By the third year the parameter is close to 95% of its steady state value of 0.1458. This can be seen in the first panel of figure 5.[27] The exercise was performed again for different values of the utility parameter σ. Four values were considered 0.5,1,2 and 3. In terms of the mechanics of the solution, the model is solved with the 14 system of rational expectations equations described in the previous section equations (3.9), (3.10), (3.7), (3.11), (3.12), (3.17), (3.18), (3.20), (3.21) and equation (3.22), modified by considering ϕ as time varying. These equations form a system to be solved for $\mu_t, \nu_t, \zeta_t, \psi_t, \upsilon_t, \omega_t, \hat{c}_t, \hat{b}_t, \Gamma_{z,t}, \hat{w}_t, \Gamma_{k,t}, \Gamma_{h,t}, \Gamma_{y,t}$ and q_t.

We want to consider in this section a credit crunch with minimal government intervention. The crunch will produce changes in many variables such as the price of bonds and the rate of growth of the economy. Looking at the government budget constraint (3.22), this means that

[26] The value for the discount factor is lower compared to traditional macro calibrations. This might be related to the simple structure assumed for the preference shock (2.2). Nevertheless positing a more realistic structure for this shock complicates the model without substantially improving the qualitative results sought for.

[27] Since the shock is unanticipated and the recovery fully predictable, this exercise was performed as an impulse-response of the system to a credit shock.

government must act to satisfy its constraint. We opt to maintain fixed the amount of normalized government debt and let government expenditure to adjust. Since government expenditure is unproductive in the economy, this case is the closest to a no-intervention case. Figure 5a shows the decrease in ϕ and its recovery, naturally the path of this parameter is independent of σ. Figure 5b shows the main result of this section, for values of σ higher than one, the credit crunch, in spite of being transitory, produces a permanent downward break in GDP's trend. Figure 5c shows how this result is related to capital accumulation as the fraction of wealth households decide to invest in capital decrease for high values of σ. Figure 5d shows that there is a recession in the economy for such values of the risk aversion parameter and Figure 5e show the deep drop in investment. Figure 5f show how consumption over GDP always falls with the crunch independently of the value of σ, in fact, it falls my more when $\sigma = 0.5$. The fact that consumption over GDP falls by more when $\sigma = 0.5$ accords with the fact that investment over GDP increases for such value, as is shown in figure 5e.

Figure 5g show that private credit over GDP falls as well independently of the value of σ. Also, independently of the value of risk aversion, the credit crunch produces a fall in the interest rate, which is shown in Figure 5h. Figure 5i show that government expenditure increases to satisfy its constraint. The fall in the interest rate on its obligations allow government to obtain more resources which are used to increase expenditure.

Figure 5j shows the behavior of the savings rate. For $\sigma = 3$ we can see that this value increase, this does not mean that more resources flow to high shock individuals because from Figure 5g we know that credit decreases. In this case wealth is decreasing so much that ζ_t increases to avoid an even sharper decrease in credit. As σ decreases the savings rate decreases as well. Figures 5k and 5l show that except for the case $\sigma = 0.5$, there is a decrease in the rate of growth of both types of capital. While the case σ less than one is not empirically relevant, is interesting

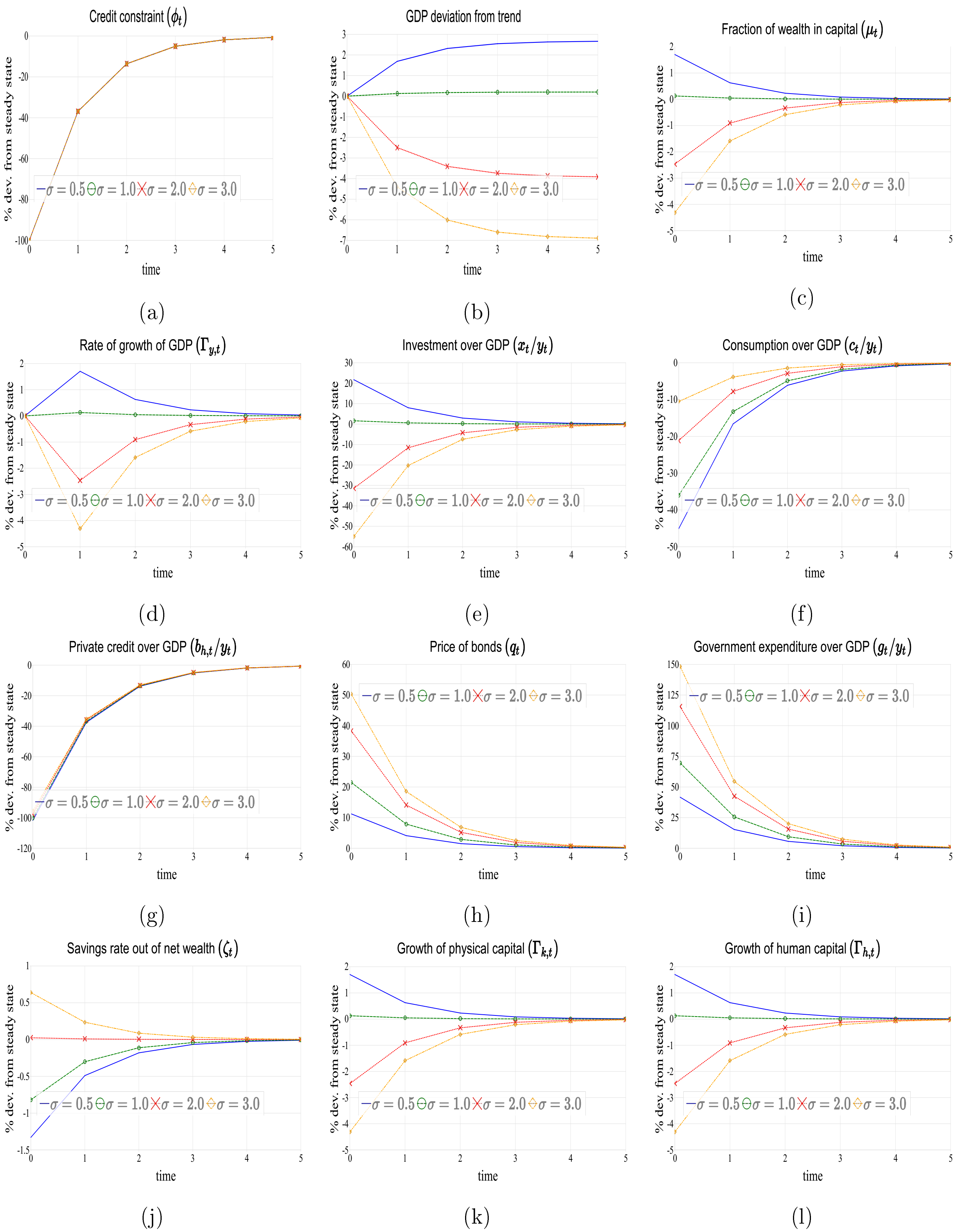


Figure 5: Some variables as function of time after the credit crunch.

to understand why there could be a positive effect for growth when the economy undergoes a credit crunch. As noted before consumption over GDP falls strongly in this case in favor of investment, which we can be seen in figures 5f and 5e. Figure 5c shows that indeed the fraction of wealth in capital increases and Figure 5d shows a boom in the economy. Furthermore Figure 5b shows that there is an upward increase in the level of GDP which is permanent. Note in Figure 5h that there is an increase in q_t even under the case $\sigma = 0.5$. Because the interest rate falls, households anticipate at the beginning of the period a lower cost to finance consumption if needed. And in case of being lenders later on they would find a low return for their savings, that is a negative income effect. When $\sigma = 0.5$ these effects become dominant and consumption over GDP decreases strongly in favor of investment, this boosts the economy in terms of growth.

4.2.1 Human capital during the Great Recession

Our model finds that a transitory credit crisis can produce a permanent break in GDP trend through the human capital channel. At first glance, this explanation may seem unlikely; however, a look at the evidence about human capital accumulation during and after the Great Recession reveals that in fact, a decline in the human capital growth rate is plausible. Both conventional wisdom and empirical evidence indicate that normally, economic downturns are associated with increases in human capital investment, especially through increases in post-secondary education enrollments.[28] During a recession, falling wages and rising unemployment lower the opportunity cost of education for many prospective students, making the choice to invest in schooling less costly. At the same time, many households face reduced income, which may reduce demand for schooling. Numerous studies have found that the price effect is dominant, and that college

[28] In the US, primary and secondary education are publicly provided for all students. While there is a private market as well at the primary and secondary levels, post-secondary costs are the most significant education-related costs faced by most households.

enrollments are counter-cyclical.[29]

However, for the Great Recession, there is still ambiguity in the literature about whether the expected counter-cyclical trend in human capital investment occurred. While some studies show evidence of increased school and college enrollments (ie:Long [2014]), there are also good arguments and some empirical evidence that overall, human capital accumulation may have slowed. The Penn World Table version 9.1 (PWT) data shows a decline in the growth rate of human capital, from about 0.4% annually in 2010, to about 0.14% annually from 2011 forward (University of Groningen). For a further discussion of the most recent version of the PWT data, see Feenstra et al. [2015]. The PWT reports human capital accumulation as average years of schooling in the population, and combines data from Barro and Lee [2013] and Cohen and Leker.

There are a number of ways in which the Great Recession was different than earlier economic downturns. Most importantly, the Great Recession was accompanied by an economy wide credit crunch. Hurd and Rohwedder [2010] identify the simultaneous shocks in the stock market, housing market, and labor market as a distinctive feature of the Great Recession. From an education perspective, Long [2014] articulates several key differences including that at the start of the recession, college costs and student debt levels were at historic highs, indicating that credit was particularly relevant at this time. Further, this recession had a substantially large and negative impact on household liquidity, and the recession coincided with a historically large cohort of graduating high school students.

Importantly, college enrollments are only one component of human capital investment; a comprehensive measure of human capital would also include health, skills, employment readiness and other components. Yagan [2019] explores the effect of the Great Recession on employment

[29]Examples include Dellas and Sakellaris [2003] for earlier recessions, and Barr and Turner [2015] and Long [2014] for the Great Recession.

in the US, finding that relatively larger unemployment shocks during the recession contributed to higher unemployment rates in subsequent years. Further, Yagan argues that this finding is partially explained by general human capital decay resulting from long periods of unemployment. This decay may occur as unemployed workers fail to keep up with evolving technologies, failure to maintain work habits such as punctuality, and an eventual choice by drop out of the labor force rather than accept lower-wage work.

There is also is substantial evidence that the great recession may have affected human capital through adverse effects on workers' health. For example, Currie et al. [2015] study maternal employment during the Great Recession, and find that increases in the unemployment rate decrease self-reported health status and increase smoking and drug use. Hurd and Rohwedder [2010] used high-frequency survey research to explore the various effects of the Great Recession on American households. In these surveys, 22 to 25% of respondents reported decreased spending in health-related categories such as doctor visits and prescription drugs. The self-reported decreases in spending on health were significantly larger than decreases in other spending categories. Burgard et al. [2013] conduct a comprehensive review of the evidence on the links between recessions and health, identifying two strands of inquiry focused on aggregate and individual effects. Especially in studies focused on individual-level effects, there are many examples of links between recession-associated events (job loss, reduction in material wealth) and negative health outcomes.

In summary, while identifying a comprehensive measure of human capital is difficult, the data and literature include several suggestive arguments that indicate a slow-down in human capital growth after the Great Recession.[30] The general conclusion of this subsection is therefore, that we cannot rule out the possibility that human capital accumulation was negatively affected

[30] Solow [2000] flagged the measurement of human capital, and understanding the relationship between human capital and growth, as critical directions for future research. Nonetheless, many of the challenges that he identified persist in the literature today.

during the Great Recession.

In the next subsection we analyze an extension, exploring the effects of government using a simple fiscal policy intervention to cope with the credit crunch.

4.2.2 Extension: Fiscal policy against the credit crunch

In response to the Great Recession, the United States enacted the American Recovery and Reinvestment Act of 2009 (ARRA), a package of economic stimulus strategies intended to offset the effects of the recession.[31] There is a substantial body of empirical work finding that stimulus policy is an effective strategy to increase consumer spending.[32] While these findings are not in line with predictions of life-cycle models, which suggest that stimulus measures, such as tax rebates, will raise household savings (rather than consumption), life-cycle models rely on perfect capital markets. The credit crunch that accompanied the Great Recession created a binding constraint in many household budgets, which could explain the consumption effects of the fiscal stimulus.[33]

However, there is limited work that we are aware of that studies the effectiveness, through the human capital channel, of fiscal policy implemented in response to a credit crisis as a tool to mitigate negative effects on growth and recovery.[34]

[31] The ARRA included a number of provisions that may have directly or indirectly affected households' human capital investment decisions. The total estimated USD 787 billion in expansionary spending allocated USD 260 billion in direct subsidies to US families, including a tax cut, an increased college tuition tax credit, and a 33-week extension of unemployment benefits. In addition, the bill included USD 138 billion in health care subsidies of various types, as well as USD 117 billion in subsidies for education, including USD 17 billion to increase funding for the Pell grant program. Most of these figures come from CBO data and estimates.

[32] Examples focused on the 2008 stimulus payments include Shapiro and Slemrod [2009], Saham et al. [2010], Parker et al. [2013], and Broda and Parker [2014]. Studies targeting earlier policy interventions include Johnson et al. [2006], Shapiro and Slemrod [1995] and Shapiro and Slemrod [2003], and Souleles [1999].

[33] See for example Zeldes [1989], Kreiner et al. [2019] and Kaplan and Violante [2014].

[34] Some investigations that study the effects of particular forms of fiscal policy on human capital accumulation without the credit dimension include: Barr and Turner [2015], Heckman et al. [1998] and Abramitzky and Lavy

While our model is not sufficient to evaluate the effectiveness of specific fiscal policy packages such as the ARRA, it can be used to broadly explore whether government intervention may counteract the negative effects of the credit crunch. Recall that in the model, government expenditure is unproductive. We study the effects of the government running a fiscal deficit by reducing taxation while the credit crunch is place. One one hand, a reduction in taxation will put more put more resources in households' hands, allowing households to devote resources to capital accumulation that cannot be financed through borrowing. On the other hand, lower taxes today means higher taxation in the future, so it is not clear that such a policy will help the economy. In a related paper, Guerrieri and Lorenzoni [2017] show that an equivalence result arises in a model with lump sum taxation. The equivalence result refers to the private and public supply of liquidity, in which there exists a sequence of lump sum taxation and government bonds that exactly offset any negative effect of a change in ϕ. In our setup, a similar result is obtained, but taxation is not lump sum. We find that increasing government debt –with reduced taxes– is beneficial to mitigate the recession and long term changes in GDP level.[35] For simplicity we tie changes in government debt to the credit friction ϕ_t:

$$\hat{b}_t = \hat{b} + \vartheta(\phi - \phi_t), \quad \vartheta > 0. \tag{4.4}$$

$\hat{b}$ is the steady state value of normalized government bonds. ϕ is the steady state value of ϕ_t, which was found to be in the calibration section 4.1 to be equal to 0.1458. When ϕ_t is lower than its steady state value, equation (4.4) says that government increases its debt. ϑ is a parameter that influence how strong is government policy. We chose ϑ to be equal to 0.8, which

[2014].

[35]The result that we find in this paper is interesting on its own. Among the features that are different from the model of Guerrieri and Lorenzoni [2017] are that we have a model of endogenous growth of human capital accumulation, the nature of the idiosyncratic shock is different, and taxation is not lump-sum.

implies that government debt over GDP duplicates at impact. Figure 6 present two scenarios, one when there is no government intervention, the same case of figure 5 under $\sigma = 2$, and the other where there is policy implemented through (4.4) to mitigate the crises.[36] Figure 6a shows that normalized government debt increases with the policy. Figure 6b show that the policy is successful to mitigate the break in GDP's trend. Figure 6c shows that the fraction of wealth devoted to capital accumulation actually decreases with the policy, but it can be shown that wealth actually increases with the policy so resources to capital accumulation have a positive impact with the policy.

Figure 6d shows how the recession is mitigated and 6e shows how investment is also less affected. Figure 6f show that consumption actually slightly increases with the policy. Figure 6g show that private credit over GDP is only slightly affected with the policy. Figure 6h shows that the interest rate does not fall by much with the policy due to the increased supply of bonds coming from the government which is also shown in Figure 6i. Figure 6j show that the savings rate increase with the policy, lenders buy government bonds and the government use the resources to lower taxes which is shown in Figure 6l, which is a reflection of the government satisfying its budget constraint (3.22). Figure 6k show how the fall in growth of human capital accumulation is mitigated due to the extra resources households obtain. Finally, Figure 6l show that after an initial reduction in taxes, the tax rate needs to increase from the second year onwards to make government debt sustainable, but this does not have a negative impact on capital accumulation and growth.

This subsection showed that fiscal policy could be beneficial to prevent short and longer term impacts on the credit crunch. Whether actual polices such as the ARRA were indeed effective in preventing deeper negative effects of the credit crunch is out of the scope of this study. If

[36]Instead of performing the analysis for different values of the risk aversion parameter, we chose to take the empirically relevant case where it is $\sigma = 2$.

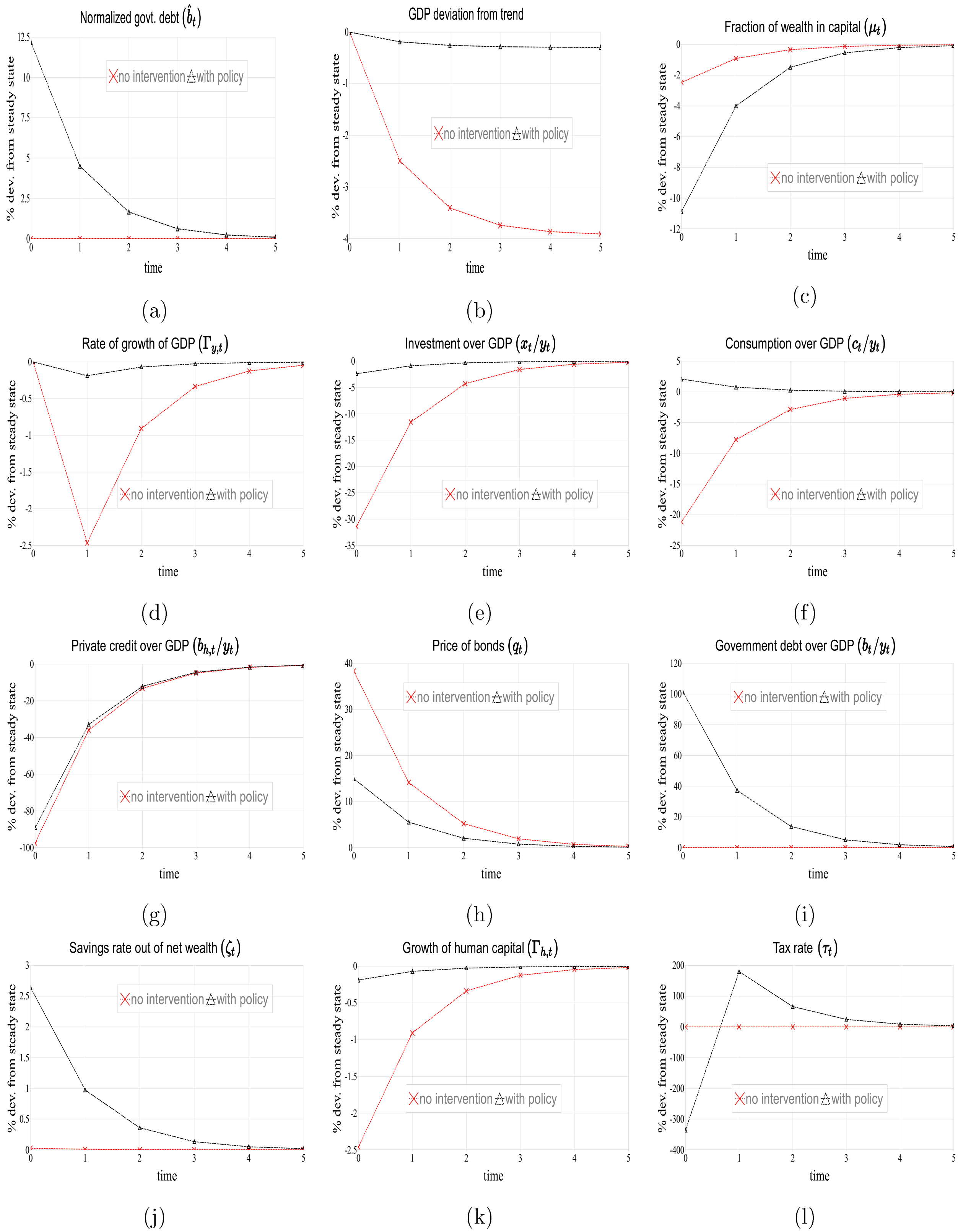


Figure 6: Some variables over time after the credit crunch with and without policy.

these policies were effective, then one would conclude, for example, that the possible break in trend portrayed in Figure 4a would have been worse absent the policy, or the channel by which such a break could have taken place is unrelated to human capital accumulation.

We now turn to analyze the consequences of productivity shocks in our model.

4.3 Productivity shocks

In this subsection we trace out the implications of productivity shocks, using the assumption in equation (2.7). We restrict ourselves to impulse response functions. The objective is to analyze the behavior of the economy under a shock that was typically analyzed in the macro literature, so we could build confidence on the generality of the model. We use $\rho = 0.9$ and the size of the shock is 1 percent of the steady state value of A. Productivity shocks are analyzed under minimal government intervention, we assume as in subsection 4.2 that to satisfy its budget constraint, the government adjust government expenditure. For comparison, we report the same variables as in Figure 5, and also reporting the results for four values of the risk aversion parameter σ, 0.5,1,2 and 3. Figure 7 show the impulse response to a positive productivity shock which is shown in Figure 7a, which of course does not depend on σ. Figure 7b show the GDP deviation from trend, which for any value of σ is upward and permanent, this contrasts with Figure 5b in which the sign of the deviation depended on the value of risk aversion.[37] Figure 7c show the fraction of wealth devoted to capital accumulation. For low values of σ it increases, for high values of σ it decreases. This behavior is related to wealth and substitution effects. The positive productivity shock causes an increase in wealth but also makes current consumption more expensive. For relative high values of risk aversion the income effect is strong and households save less of

[37] In Figure 7, we chose to report the responses for 5 years after the shock, to make them comparable to Figure 5. Extending the time period after the crunch in Figure 7b reveals that the effect for trend in GDP is permanent.

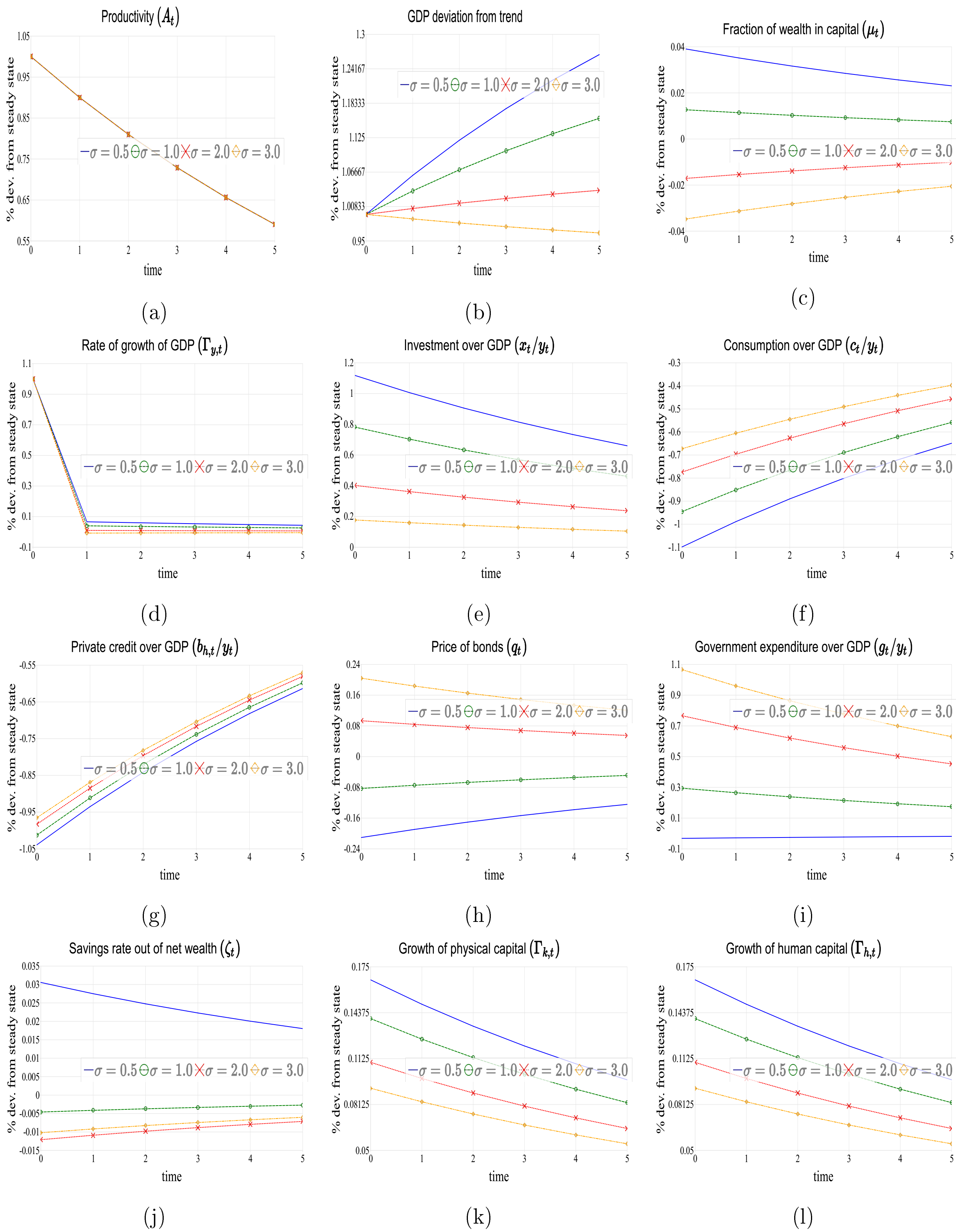


Figure 7: Impulse response functions to a productivity shock.

their wealth for the future. Figure 7d show that the positive productivity shock causes an expansion in the economy independently of the risk aversion parameter. Figures 7e and 7f show investment and consumption over GDP respectively. Again we see that for low values of the risk aversion parameter, the substitution effect is strong and investment raises more and consumption decreases by more as compared to high values of σ. Figure 7g show that credit over GDP falls with a positive productivity shock independently of the value of σ. Figure 7h show the behavior of the price of bonds. Supply of bonds increases with the shock because wealth increases and demand of funds is determined by a fraction ϕ of wealth, see equation (2.5), this induces q_t to drop, but demand of bonds have a different behavior depending on the value of σ. For high values of σ, specifically above one, demand of bonds increases strongly and the equilibrium q_t increases. Savers with high values of σ want to avoid strong swings in their consumption. One one hand at the beginning of the period they choose to reduce their fraction of wealth in capital, to insure themselves for the possibility of facing a high shock later on and so have ample funds. When they actually face a low preference shock they use these funds reserved from the beginning of the period in large part to save for the future and that is why they demand more bonds and its price increases. Figure 7i show that normalized government expenditure always increases with the positive productivity shock, even when q_t decreases the growth effect is strong enough that the government obtain more resources issuing debt and government expenditure increase to satisfy its budget constraint. Figure 7j show that the savings rate out of net wealth decreases except for the case $\sigma = 0.5$. For this value household allow for large swings in their consumption, under the strong substitution effect, inducing them to save more for the future. Finally, Figures 7k and 7l show how savings in capital increase, in higher magnitude for low values of risk aversion when the substitution effect is strong.

5 Conclusions

We develop a model of credit constraints, where human capital accumulation is the basis of endogenous growth. While the model was not built to make exact quantitative predictions, it is successful in providing a rationale to some empirical findings in the literature concerning finance and growth, and an explanation for transitory financial disruptions in producing persistent negative consequences in the macroeconomy.

First, we investigate the relationship between finance and growth and find that our model delivers a non-monotonic relationship between the two that qualitatively mimics the inverted U shape relationship found in recent empirical work. In addition we derived several testable implications for growth and finance that we think are valuable given the relative scarcity of models that combine finance, growth and human capital.

Second, we explore the possible long-term effects of transitory financial disruptions. As discussed in the text, a permanent break in GDP trend appears to have occurred during the Great Recession of 2008, despite the fact that the financial disruption was itself transitory and short lived. The model we developed in this paper is capable of producing a permanent break in trend GDP caused by a transitory credit crunch, and therefore we provide a plausible explanation for this phenomena based on the human capital channel.

A Appendix

In this Appendix we go into the details of the solution of the model. In subsection A.1 we go over the details of the method of solution of the model, which relies on a Guess-and-Verify strategy. In subsection A.2 we show how the model displays the aggregate identity that total GDP equals the sum of consumption, investment and government expenditure. Finally in subsection A.3 we present the equations of the model in steady state.

A.1 Solution of the model

We tackle the solution of the problem in (3.3) by first working out the inner problem, conditional on a given value for θ_i. Let us define:

$$\mathcal{G}_t(w_{i,t};\theta_{j,t}) = \max_{c_{i,t},b_{i,t}} \left[\theta_{j,t} u(c_{i,t}) + \beta \mathbb{E}_t \mathcal{V}_{t+1}(w_{i,t+1})\right] \tag{A.1a}$$

As the value function for the inner problem in (3.3). Using the definition of $w_{i,t}$ in (3.2), shifting forward one period we have: $w_{i,t+1} = \left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right] z_{i,t+1} + b_{i,t+1}$. Then using (3.5), problem (A.1a) is solved subject to the constraints:

$$q_t b_{i,t+1} + c_{i,t} = (1-\mu_t) w_{i,t}, \quad w_{i,t+1} = \left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right] \mu_t w_{i,t} + b_{i,t+1} \tag{A.1b}$$

and subject to the liquidity constraint:

$$b_{i,t+1} \geq -\phi w_{i,t} \tag{A.1c}$$

We now solve for the policy functions depending on whether the household receive a low or a high preference shock.

Working with low shock individuals

For all those individuals whose $\theta_i = \theta_\ell$, we assume that they are not credit constrained. Then their optimization carries under constraints (A.1b) only. These individuals have resources given by $(1-\mu_t)w_{i,t}$ at this stage. Because $b_{i,t+1}$ is by assumption non-negative for them, consumption must be a fraction, say $(1-\zeta_t)$, of current resources.[38] Using this fact, equations (A.1b) can be written as:

$$c^\ell_{i,t} = (1-\zeta_t)(1-\mu_t)w_{i,t}, \quad q_t b^\ell_{i,t+1} = \zeta_t(1-\mu_t)w_{i,t} \tag{A.1d}$$

and:

$$w^\ell_{i,t+1} = \left\{ \left[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})\right]\mu_t + \frac{\zeta_t}{q_t}(1-\mu_t) \right\} w_{i,t} \tag{A.1e}$$

Where a supraindex ℓ over individual consumption, bonds and wealth, denotes the values of these variables for an individual i facing a low shock. For these low shock individuals, problem (A.1a) amounts to:

$$\mathcal{G}_t(w_{i,t};\theta_\ell) = \max_{\zeta_t} \left\{ \theta_\ell u(c^\ell_{i,t}) + \beta \mathbb{E}_t \mathcal{V}_{t+1}\left(w^\ell_{i,t+1}\right) \right\} \tag{A.1f}$$

The FOC for the problem is:

$$\theta_\ell u'(c^\ell_{i,t})(1-\mu_t)w_{i,t} = \beta \mathbb{E}_t \mathcal{V}'_{t+1}\left(w^\ell_{i,t+1}\right) \frac{1-\mu_t}{q_t} w_{i,t} \tag{A.1g}$$

Equation (A.1g) states that individuals optimizing will equate the marginal cost of differing

[38] $\zeta_t > 0$ is a value to be determined and amounts to the savings rate out of net wealth, the savings rate net of expenditure on both types of capital.

consumption for the future with the marginal benefit of the extra savings. Note that the higher the interest rate on bonds (the lower q), individuals have a higher marginal benefit of savings and therefore tend to increase savings my more. Using the functional form for the utility function plus the guess for the value function in (3.4), we get equation (3.7) in the main text.

Working with high shock individuals

For all those individuals whose $\theta_i = \theta_h$, we assume that they are credit constrained, and therefore their policy functions are:

$$c^h_{i,t} = (q_t \phi_t + 1 - \mu_t) w_{i,t}, \quad b^h_{i,t} = -\phi w_{i,t} \tag{A.1h}$$

With next period total resources:

$$w^h_{i,t+1} = \left\{ \left[r_{k,t+1} \nu_{t+1} + r_{h,t+1} (1 - \nu_{t+1}) \right] \mu_t - \phi \right\} w_{i,t} \tag{A.1i}$$

Equation (A.1a) for high shock individuals is:

$$\mathcal{G}_t(w_{i,t}; \theta_h) = \left\{ \theta_h u(c^h_{i,t}) + \beta \mathbb{E}_t \mathcal{V}_{t+1} \left(w^h_{i,t+1} \right) \right\} \tag{A.1j}$$

Given the characterization of the policy functions just obtained, we tackle the outer problem in (3.3), working out the beginning of period decisions.

Working with beginning of period decisions

Households need to decide at the beginning of each period, the total amount of capital to accumulate for next period ($z_{i,t+1}$) and how it is divided among physical and human capital (ν_{t+1}), because of (3.5), this is the same as choosing μ_t and ν_{t+1}. From the Bellman equation

(3.3) we have, :

$$\mathcal{V}_t(w_{i,t}) = \max_{\mu_t,\nu_{t+1}} \left[\pi_\ell \mathcal{G}_t(w_{i,t};\theta_\ell) + \pi_h \mathcal{G}_t(w_{i,t};\theta_h)\right] \tag{A.1k}$$

Where $\mathcal{G}_t(w_{i,t};\theta_\ell)$ and $\mathcal{G}_t(w_{i,t};\theta_h)$ are given by (A.1f) and (A.1j) respectively.

Note that individuals must take into account how the net savings rate ζ_t changes when evaluating different values for μ_t and ν_{t+1}. To find these relationships, we use the Implicit Function Theorem in (3.7) to find:

$$\frac{d\zeta_t}{d\mu_t} = -\frac{\frac{\theta_\ell q_t(1-\zeta_t)}{[(1-\zeta_t)(1-\mu_t)]^{\sigma+1}} + \beta\mathbb{E}_t \frac{\psi_{t+1}\left[r_{k,t+1}\nu_{t+1}+r_{h,t+1}(1-\nu_{t+1})-\frac{\zeta_t}{q_t}\right]}{\left\{\left[r_{k,t+1}\nu_{t+1}+r_{h,t+1}(1-\nu_{t+1})\right]\mu_t+\frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{\sigma+1}}}{\frac{\theta_\ell q_t(1-\mu_t)}{[(1-\zeta_t)(1-\mu_t)]^{\sigma+1}} + \beta\mathbb{E}_t \frac{\psi_{t+1}(1-\mu_t)/q_t}{\left\{\left[r_{k,t+1}\nu_{t+1}+r_{h,t+1}(1-\nu_{t+1})\right]\mu_t+\frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{\sigma+1}}} \tag{A.1l}$$

$$\frac{d\zeta_t}{d\nu_{t+1}} = -\frac{\beta\mathbb{E}_t \frac{\psi_{t+1}\left(r_{k,t+1}-r_{h,t+1}\right)}{\left\{\left[r_{k,t+1}\nu_{t+1}+r_{h,t+1}(1-\nu_{t+1})\right]\mu_t+\frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{\sigma+1}}}{\frac{\theta_\ell q_t(1-\mu_t)}{[(1-\zeta_t)(1-\mu_t)]^{\sigma+1}} + \beta\mathbb{E}_t \frac{\psi_{t+1}(1-\mu_t)/q_t}{\left\{\left[r_{k,t+1}\nu_{t+1}+r_{h,t+1}(1-\nu_{t+1})\right]\mu_t+\frac{\zeta_t}{q_t}(1-\mu_t)\right\}^{\sigma+1}}} \tag{A.1m}$$

Then, the FOCs for μ and ν_{t+1} in (A.1k) is given by equations (3.9) and (3.10) in the text respectively.

We still need to determine the value of ψ_t. The envelope condition applied to (A.1k), using (A.1f) and (A.1j):

$$\mathcal{V}'_t(w_{i,t}) = \pi_\ell \mathcal{G}'_t(w_{i,t};\theta_\ell) + \pi_h \mathcal{G}'_t(w_{i,t};\theta_h) \tag{A.1n}$$

where:

$$\mathcal{G}'_t(w_{i,t};\theta_\ell) = \theta_\ell u'(c^\ell_{i,t})(1-\zeta_t)(1-\mu_t)+\beta\mathbb{E}_t\mathcal{V}'_{t+1}\left(w^\ell_{i,t}\right)\left\{\left[r_{k,t+1}\nu_{t+1}+r_{h,t+1}(1-\nu_{t+1})\right]\mu_t + \frac{\zeta_t}{q_t}(1-\mu_t)\right\} \tag{A.1o}$$

and:

$$\mathcal{G}_t'(w_{i,t};\theta_h) = \theta_h u'(c_{i,t}^h)(q_t\phi_t + 1 - \mu_t) + \beta\mathbb{E}_t\mathcal{V}_{t+1}'\left(w_{i,t}^h\right)\{[r_{k,t+1}\nu_{t+1} + r_{h,t+1}(1-\nu_{t+1})]\,\mu_t - \phi_t\} \tag{A.1p}$$

Using the utility function and the guess (3.4), equation (A.1n) translates to equation (3.11) in the text. Note that as a matter of verification step of the guess-and-verify method, replacing the guess in equation (A.1k) and using (A.1f) and (A.1j), we get exactly the equation (3.11).

A.2 The national income identity

In this subsection we show how the model satisfies the national income identity that output equals consumption plus investment plus government expenditures.

Aggregating the budget constraint (2.3), we have:

$$z_{t+1} + q_t b_{t+1} + c_t = [r_{k,t}\nu_t + r_{h,t}(1-\nu_t)]z_t + b_t \tag{A.2a}$$

Replacing the definitions of $r_{k,t}$ and $r_{h,t}$ from equations (2.4), this equation is:

$$z_{t+1} + q_t b_{t+1} + c_t = (1-\tau_t)[\upsilon\nu_t + \omega_t(1-\nu_t)]z_t + b_t \tag{A.2b}$$

Note also that the government budget constraint (2.8) can be written as:

$$b_t + g_t = \tau_t[\upsilon_t\nu_t + \omega_t(1-\nu_t)]z_t + q_t b_{t+1} \tag{A.2c}$$

Replacing (A.2c) into the budget constraint (A.2b), we have:

$$z_{t+1} - (1-\delta)z_t + g_t + c_t = [\upsilon_t \nu_t + \omega_t(1-\nu_t)]z_t \tag{A.2d}$$

This equation is showing that investment plus government expenditure plus consumption equals to a term involving the price of inputs, ν_t and z_t. This term in the RHS of equation (A.2d) must be equal, of course, to output. Let us show that this is indeed the case. The term in brackets in the RHS in (A.2d) can be written, using equations (3.12):

$$\upsilon_t \nu_t + \omega_t(1-\nu_t) = A_t \alpha \left(\frac{1-\nu_t}{\nu_t}\right)^{1-\alpha} \nu_t + A_t(1-\alpha)\left(\frac{\nu_t}{1-\nu_t}\right)^{\alpha}(1-\nu_t) = A_t \nu_t^{\alpha}(1-\nu_t)^{1-\alpha} \tag{A.2e}$$

Now, equation (2.6), the production function can be written as:

$$y_t = A_t \left(\frac{k_t}{k_t+h_t} z_t\right)^{\alpha} \left(\frac{h_t}{k_t+h_t} z_t\right)^{1-\alpha} = A_t \nu_t^{\alpha} z_t^{\alpha} (1-\nu_t)^{1-\alpha} z_t^{1-\alpha} = A_t \nu_t^{\alpha}(1-\nu_t)^{1-\alpha} z_t \tag{A.2f}$$

Which shows that the RHS of (A.2d) is indeed aggregate output.

A.3 The model in steady state

The long run analysis of the model is undertaken by analyzing the steady state. We start by noticing that because the fraction of total capital devoted to physical capital is chosen at the beginning of the period, returns of both capitals must be equal in equilibrium, that is $r_k = r_h \equiv r$ must hold. Note that in this case (A.1m) in steady state implies that $d\zeta/d\nu = 0$ and that the FOC with respect to ν in steady state from equation (3.10) holds.[39] Given the

[39]Equation (3.10) is composed of three terms. The first term is zero because $d\zeta/d\nu = 0$. The second term is composed of the product of two terms, the second of which is zero because $r_k = r_h$ and because $d\zeta/d\nu = 0$. Finally, the third term is zero because $r_k = r_h$.

definition of both r_k and r_h in steady state and the fact that both taxation and depreciation are assumed to be equal for both types of capital, equations in (2.4) imply that $\upsilon = \omega$. Equations (3.12) can then be solved for ν and we obtain $\nu = \alpha$. This in turn implies, using any of the equations in (3.12), that $\upsilon = \omega = A(1-\alpha)^{1-\alpha}\alpha^{\alpha}$. Using any of the equations in (2.4) we have $r = 1 - \delta + (1-\tau)A(1-\alpha)^{1-\alpha}\alpha^{\alpha}$.

With the results derived above, the FOC with respect to μ from (3.9) is written as:

$$\begin{aligned} - \quad & \pi_\ell \theta_\ell \left[(1-\zeta)(1-\mu)\right]^{-\sigma} \left[\frac{d\zeta}{d\mu}(1-\mu) + 1 - \zeta\right] + \pi_\ell \beta \psi \left\{ r\mu + \frac{\zeta}{q}(1-\mu) \right\}^{-\sigma} \\ & \left\{ r + \frac{1}{q}\left[\frac{d\zeta}{d\mu}(1-\mu) - \zeta\right]\right\} - \pi_h \theta_h (q\phi + 1 - \mu)^{-\sigma} + \pi_h \beta \psi \left\{ r\mu - \phi \right\}^{-\sigma} r = 0 \quad \text{(A.3a)} \end{aligned}$$

The equation determining the optimality of ζ, equation (3.7) in steady state is:

$$\theta_\ell \left[(1-\zeta)(1-\mu)\right]^{-\sigma} q = \beta \psi \left\{ r\mu + \frac{\zeta}{q}(1-\mu) \right\}^{-\sigma} \tag{A.3b}$$

The equation determining ψ, equation (3.11) in steady state:

$$\psi = \pi_\ell \theta_\ell [(1-\zeta)(1-\mu)]^{1-\sigma} + \pi_\ell \beta \psi \left\{ r\mu + \frac{\zeta}{q}(1-\mu) \right\}^{1-\sigma} + \pi_h \theta_h (q\phi + 1 - \mu)^{1-\sigma} + \pi_h \beta \psi \left\{ r\mu - \phi \right\}^{1-\sigma} \tag{A.3c}$$

Next, the second equation in (3.20) in steady state gives $\Gamma_k = \Gamma_z$, while from the first equation we get $\Gamma \equiv \Gamma_z = \Gamma_k = \Gamma_h$. Note also from equation (3.21) that $\Gamma_y = \Gamma$, hence human capital, physical capital and output all grow at the same rate in steady state.

From equations (3.17) in steady state we have:

$$\hat{c} = [(1-\zeta)(1-\mu)\pi_\ell + (q\phi + 1 - \mu)\pi_h]\hat{w},\ \Gamma\hat{b} = \left[\frac{\zeta}{q}(1-\mu)\pi_\ell - \phi\pi_h\right]\hat{w},\ \Gamma = \mu\hat{w} \qquad \text{(A.3d)}$$

with normalized wealth from (3.18) equal to $\hat{w} = r + \hat{b}$. Finally normalized government budget constraint from (3.22) in steady state:

$$\hat{b} + \hat{g} = \tau\omega + q\hat{b}\Gamma \qquad \text{(A.3e)}$$

To solve the model in steady state we need to solve for the non-linear system composed of equations (A.3a) to (A.3e) for the seven unknowns $\Gamma, \zeta, \mu, \psi, \hat{c}, q$ and $\hat{b}$, if normalized government expenditure is assumed exogenous. The system does not have a closed form solution and needs to be solved using numerical methods.

References

Ran Abramitzky and Victor Lavy. How responsive is investment in schooling to changes in redistributive policies and in returns? *Econometrica*, 82(4):1241–1272, 2014.

Abdulsalam Abubakar, Salina HJ Kassim, and Mohammed B Yusoff. Financial development, human capital accumulation and economic growth: empirical evidence from the economic community of west african states (ecowas). *Procedia-Social and Behavioral Sciences*, 172: 96–103, 2015.

Jean-Louis Arcand, Enrico Berkes, and Ugo Panizza. Too much finance? *Journal of Economic Growth*, 20(2):105–148, 2015.

Debasis Bandyopadhyay, Ian King, and Xueli Tang. Human capital misallocation, redistributive policies, and tfp. *Journal of Macroeconomics*, 60:309–324, 2019.

Andrew Barr and Sarah Turner. Out of work and into school: Labor market policies and college enrollment during the great recession. *Journal of Public Economics*, 124:63–73, 2015.

Robert J Barro and Jong Wha Lee. A new data set of educational attainment in the world, 1950–2010. *Journal of Development Economics*, 104:184–198, 2013.

Thorsten Beck, Berrak Buyukkarabacak, Felix Rioja, and Neven Valev. Who gets the credit? and does it matter? household vs. firm lending across countries. *The B.E. Journal of Macroeconomics*, 12(1 (Contributions), Article 2), 2012.

Christian Broda and Jonathan A. Parker. The economic stimulus payments of 2008 and the aggregate demand for consumption. *Journal of Monetary Economics*, 68((S)):S20–36, 2014.

Sarah A Burgard, Jennifer A Ailshire, and Lucie Kalousova. The great recession and health:

People, populations, and disparities. *The Annals of the American Academy of Political and Social Science*, 650(1):194–213, 2013.

Elizabeth M. Caucutt and Lance Lochner. Early and late human capital investments, borrowing constraints, and the family. *Journal of Political Economy*, 128(3):1065–1147, 2020.

Daniel Cohen and Laura Leker. Health and education: Another look with the proper data. *CEPR Discussion Paper*, DP 9940.

Carol Corrado, Charles Hulten, and Daniel Sichel. Intangible capital and u.s. economic growth. *The Review of Income and Wealth*, 55(3):661–685, 2009.

Janet Currie, Valentina Duque, and Irwin Garfinkel. The Great Recession and Mothers' Health. *The Economic Journal*, 125(588):F311–F346, 11 2015.

Harris Dellas and Plutarchos Sakellaris. On the cyclicality of schooling: Theory and evidence. *Oxford Economic Papers*, 55(1):148–172, 2003.

Alun Evans, Christopher Green, and Victor Murinde. Human capital and financial development in economic growth: new evidence using the translog production function. *International Journal of Finance & Economics*, 7(2):123–140, 2002.

Robert C Feenstra, Robert Inklaar, and Marcel P Timmer. The next generation of the penn world table. *American Economic Review*, 105(10):3150–82, 2015.

John Fernald, Robert Hall, James Stock, and Mark Watson. The disappointing recovery of output after 2009. *Brookings Papers on Economic Activity*, 2017(1):1–81, 2017.

Piero Gottardi, Atsushi Kajii, and Tomoyuki Nakajima. Human capital risk and economic growth. *American Economic Review*, 105(11):3443–3470, 2015.

Veronica Guerrieri and Guido Lorenzoni. Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467, 2017.

Pablo Guerron-Quintana and Ryo Jinnai. Financial frictions, trends, and the great recession. *Quantitative Economics*, 10(2):735–773, 2019.

Mobolaji Hakeem. Banking development, human capital and economic growth in sub-saharan africa (ssa). *Journal of Economic Studies*, 37(5):557–577, 2010.

James J. Heckman, Lance Lochner, and Christopher Taber. Tax policy and human-capital formation. *American Economic Review*, 88(2):293–297, 1998.

Yu-Fan Huang and Sui Luo. Potential output and inflation dynamics after the great recession. *Empirical Economics*, 55(2):495–517, 2018.

Michael D. Hurd and Susann Rohwedder. Effects of the financial crisis and great recession on american households. *NBER Working Paper*, 16407, 2010.

Tulio Japelli. Who is credit constrained in the u.s. economy? *Quarterly Journal of Economics*, 105(1):219–234, 1990.

David S. Johnson, Jonathan A. Parker, and Nicholas S. Souleles. Household expenditure and the income tax rebates of 2001. *American Economic Review*, 96(5):1589–1610, 2006.

Greg Kaplan and Giovanni L. Violante. A model of the consumption response to fiscal stimulus payments. *Econometrica*, 82(4):1199–1239, 2014.

Cuneyt Kilic and Burcu Ozcan. The impact of financial development on human capital: Evidence from emerging market economies. *International Journal of Economics and Financial Issues*, 8(1):258–267, 2018.

Robert King and Ross Levine. Finance and growth: Schumpeter might be right. *Quarterly Journal of Economics*, 108(3):717–737, 1993.

Tom Krebs. Human capital risk and economic growth. *The Quarterly Journal of Economics*, 118(2):709–744, 2003.

Claus T. Kreiner, David D. Lassen, and Søren Leth-Petersen. Liquidity constraint tightness and consumer responses to fiscal stimulus policy. *American Economic Journal: Economic Policy*, 11(1):351–379, 2019.

Siong Hook Law and Nirvikar Singh. Does too much finance harm economic growth? *Journal of Banking & Finance*, 41:36–44, 2014.

Ross Levine. Finance and growth: Theory and evidence. *Handbook of Economic Growth*, 1(Part A):865–934, 2005.

Lance J. Lochner and Alex Monge-Naranjo. The nature of credit constraints and human capital. *American Economic Review*, 6(101):2487–2529, 2011.

Lance J. Lochner and Alex Monge-Naranjo. Credit constraints in education. *Annual Review of Economics*, 4:225–256, 2012.

Bridget Terry Long. The financial crisis and college enrollment: How have students and their families responded? *How the Financial Crisis and Great Recession Affected Higher Education, Jeffrey R. Brown and Caroline M. Hoxby, eds.*, pages 209–233, 2014.

Robert Lucas. Equilibrium in a pure currency economy. *Economic Inquiry*, 18(2):203–220, 1980.

Robert Lucas. On efficiency and distribution. *Economic Journal*, 102(411):233–247, 1992.

Robert Lucas. On the mechanics of economic development. *Journal of Monetary Economics*, 22(1):3–42, 1998.

Gregory Mankiw, David Romer, and David Weil. A contribution to the empirics of economic growth. *Quarterly Journal of Economics*, (107):407–437, 2002.

Kazuo Mino. A simple model of endogenous growth with financial frictions and firm heterogeneity. *Economics Letters*, 127:20–23, 2015.

Quoc Hung Nguyen. Financial deepening in a two-sector endogenous growth model with productivity heterogeneity. *The BE Journal of Macroeconomics*, 2019.

Mutiu Oyinlola and Abdulfatai Adedeji. Human capital, financial sector development and inclusive growth in sub-saharan africa. *Economic Change and Restructuring*, 52(1):43–66, 2019.

Jonathan A. Parker, Nicholas S. Souleles, David S. Johnson, and Robert McClelland. Consumer spending and the economic stimulus payments of 2008. *American Economic Review*, 103(6): 2530–2553, 2013.

Werner Roeger, Janos Varga, and Jan In't Veld. Modelling the lisbon strategy: analysing policies to promote knowledge investment with an endogenous growth model. *Comparative Economic Studies*, 51(4):520–539, 2009.

Paul Romer. Endogenous technological change. *Journal of Political Economy*, 98(5):S71–S102, 1990.

Peter Rousseau and Paul Wachtel. What is happening to the impact of financial deepening on economic growth? *Economic Inquiry*, 49(1):276–288, 2009.

Claudia R. Saham, Matthew D. Shapiro, and Joel Slemrod. Household response to the 2008 tax rebate: Survey evidence and aggregate implications. *Tax Policy and the Economy*, 24(1): 69–110, 2010.

Matthew D. Shapiro and Joel Slemrod. Consumer response to the timing of income: Evidence from a change in tax withholding. *American Economic Review*, 85(1):274–283, 1995.

Matthew D. Shapiro and Joel Slemrod. Consumer response to tax rebates. *American Economic Review*, 93(1):381–396, 2003.

Matthew D. Shapiro and Joel Slemrod. Did the 2008 tax rebates stimulate spending? *American Economic Review*, 99(2):374–379, 2009.

Robert M Solow. Toward a macroeconomics of the medium run. *Journal of Economic Perspectives*, 14(1):151–158, 2000.

Nicholas S. Souleles. The response of household consumption to income tax refunds. *American Economic Review*, 89(4):947–958, 1999.

Yi Wen. Money, liquidity and welfare. *European Economic Review*, 76:1–24, 2015.

Danny Yagan. Employment hysteresis from the great recession. *Journal of Political Economy*, 127(5):2505–2558, 2019.

Stephen P. Zeldes. Consumption and liquidity constraints: An empirical investigation. *Journal of Political Economy*, 97(2):305–346, 1989.