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Abstract

There is by now reasonable evidence that supports the notion of a trend break in the US GDP since the Great Recession. To explain this phenomenon, I construct a version of the Lucas endogenous growth model, amplified with financial frictions and financial disruptions in the firms' sector. I then show how a *transitory* liquidity crunch is capable, at least qualitatively, of producing a similar pattern of a *persistent* downward shift in the GDP trend as one could infer happened in the US since 2008. The main mechanism by which such a result is found relies on workers' decisions on providing labor to firms versus accumulating human capital. I show that a transitory liquidity crunch reduces the demand of labor. Workers anticipating a phase of depressed wages make the decision of accumulating more human capital in the short run, thereby reducing labor supply to firms. In the long run, however, incentivized by a strong recovery, workers decrease human capital accumulation and increase labor supply. Under plausible parametrizations of the model, this situation produces a net effect of a decrease in overall productivity that *permanently* reduces the trend at which the economy was growing prior to the crisis.

Keywords: endogenous growth, liquidity crises, human capital.

JEL Classification: O4,G01,E44.

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1 Introduction

In one of the most cited papers in the profession, Lucas [1988], ruling out from the outset the importance of financial frictions for growth, states that: *"In general, I believe that the importance of financial matters is very badly over-stressed in popular and even much professional discussion and so am not inclined to be apologetic for going to the other extreme"*. In the present paper, I extend Lucas' model, allowing for the existence of financial frictions in the firm's side, and perhaps surprisingly, even this expanded theoretical model delivers an irrelevant result for finance in long-run growth. However, I also find that transitory financial disruptions can produce permanent effects on the trend level of GDP.

Figure 1 shows the logarithm of US GDP since 1947. I have added an estimation of the GDP trend until the fourth quarter of 2008 and extrapolated it beyond that period. It is clear that there are elements to conclude a break in the GDP trend.[1]

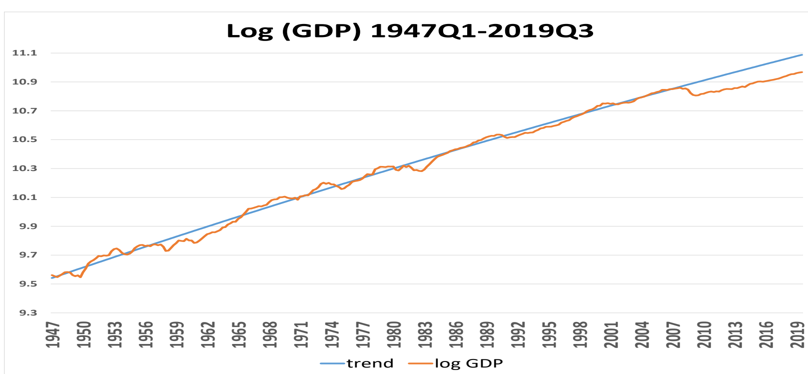


Figure 1: Log of GDP 1947Q1-2019Q3. Quadratic trend 1947Q1-2008Q4, extrapolated.

[1] However, decomposing a series to extract a cycle and a trend is always a tricky business. For example, Fernald et al. [2017] argue, using Okun's law, that output per capita had started to decrease prior to 2008. On the other hand, Huang and Luo [2018] find econometric evidence that potential output indeed declined substantially after the Great Recession. Using a structural, general equilibrium, growth model, Guerron-Quintana and Jinnai [2019] also identify a break in the GDP trend, emphasizing the R&D channel.

The figure shows that while the rate of growth of the economy appears to be approximately the same as prior to the liquidity crunch of 2008, the level is below what it should have been based on the economy's past performance. In as much as there has been a transitory financial disruption, Lucas' theory appears correct, when contrasted with the data, that the rate of growth of the economy seems to be unaffected by financial factors. However, what about the level? The model proposed in this paper shows that the level can be permanently affected by a transitory financial disruption, therefore providing a possible explanation of the inferred break in trend in the data.[2]

I expand Lucas's model by enriching the production sector of the economy. I allow for entrepreneurs to produce output in the economy and make them heterogenous in their investment opportunities so that they can trade in financial assets. I also introduce financial frictions in a fashion that was successfully implemented in the literature of financial fluctuations and the macroeconomy; see, for example, Kiyotaki and Moore [2019]; Shi [2015] and Guerron-Quintana and Jinnai [2019]. These entrepreneurs hire labor from workers who are a separate group of individuals that are not affected directly by developments in the financial sector but accumulate human capital, which is the engine of growth in this economy. I then show that under some plausible parametrizations of the model, a *transitory* financial disruption can produce a *transitory* contraction—as measured by the rate of growth—but a *permanent* downward shift in the GDP trend.

The basic mechanism by which the result is found relies on the interaction among investment in physical capital, the asset market, the labor market—which is assumed to be Walrasian—and human capital. In the short run, the liquidity crunch produces less financing of investment with the consequent reduction in labor demand and a detrimental effect on wages. Under plausible parametrizations of the model, workers anticipating a period of depressed wage make the decision

[2]That the financial crisis of 2008 could produce persistent effects on GDP has been studied at a statistical level, not only for the US but also for many advanced countries; see, for example, Romer and Romer [2017] and Barnichon et al. [2020].

of accumulating more human capital in the short run, thereby reducing labor supply to firms. Afterwards, when the liquidity crunch subsides, entrepreneurs' capital creation is strong, pushing up demand of labor and wages. This situation gives incentives to workers to cut down on their human capital accumulation, which has a detrimental effect on productivity in the economy. Therefore, despite a strong recovery in physical capital accumulation, the economy does not recover enough to put GDP in the trend at which it was growing prior to the crunch.

The rest of the document is organized as follows. Section 2 presents the main model. Section 3 presents the dynamic analysis of a liquidity crunch; this section also includes an extension of the model expanding the portfolio assets of entrepreneurs. Section 4 concludes. The Appendix contains several details and derivations of results.

2 The Model

Overview

The model consists of two groups of individuals, namely, entrepreneurs and workers. Entrepreneurs hire workers and use capital to produce the consumption good of the economy. They also have the possibility to invest, such that with probability π in each period, they receive an investment opportunity. Because there is heterogeneity among this group of agents, there will be credit in equilibrium that will arise in the form of financial claims over capital. Modeling of the entrepreneurial side is done in a similar vein as in Kiyotaki and Moore [2019].[3]

By assumption, workers do not participate in the financial asset market.[4] Workers only have a

[3]Other contributions that took a similar framework to model credit and financial frictions include Shi [2015] and Guerron-Quintana and Jinnai [2019].

[4]Kiyotaki and Moore [2019] show in a similar environment that if workers have GHH preferences, then they

time endowment and a technology to transform time to human capital. They decide each period how many hours to offer as labor to entrepreneurs or acquire more human capital; this part of the model is most similar to Lucas [1988]. I explain now the main features of the model, while Appendix A contains more detailed derivations.

Entrepreneurs

There is a measure one of entrepreneurs; time is discrete $t = \{0, 1, 2, ..., \infty\}$. They seek to maximize:[5]

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(c_{e,t+s}), \quad 0 < \beta < 1, \quad u(c_{e,t}) = \frac{c_{e,t}^{1-\sigma}}{1-\sigma}. \tag{2.1}$$

where $c_{e,t}$ is the current entrepreneur's consumption, β is the discount factor and $u(\cdot)$ is the period utility function that has risk aversion parameter σ. The expectation operator $\mathbb{E}_t$ refers to an uninsurable idiosyncratic risk of the following type. All entrepreneurs in each period face the possibility of creating capital with probability π; when an agent has the chance, his or her status will be denoted $z = i$ for investor. When an agent does not have the chance, his or her status will be denoted $z = s$ for saver, as he or she will be financing partially capital investments by investors. Their status is i.i.d. over time and across individuals.[6] Capital accumulation then

would chose not to participate in the asset market. In our formulation, we do not impose such preferences but use the standard CRRA specification; it turns out that the degree of risk aversion under these preferences has important implications for the results of the paper regarding the level of GDP attained after the liquidity crunch. Even though imposing the constraint that workers do not participate in the asset market is a stark assumption, it simplifies the analysis considerably, and it is defendable to some extent because a substantial fraction of households indeed do not have access to financial markets.

[5] To ease on notation, I avoid using subindexes to denote individual's quantities. For most of the variables to be used in the paper, lowercase letters will denote individual variables and uppercase letters will denote aggregates. I will also employ recursive notation, in which a prime over a variable will denote the next period value of the variable.

[6] The model and method of solution admits the introduction of aggregate shocks, either productivity shocks or liquidity shocks. I will disregard productivity shocks as there is little evidence that such a shock would be the cause of the Recession of 2008. Recurrent liquidity shocks will also be disregarded, as the liquidity crunch of 2008 can be better viewed as a one-time, unexpected event, which is the modeling assumption that I will use in this paper.

satisfies:

$$k' = \begin{cases} (1-\delta_k)k + x & \text{if} \quad z = i \\ (1-\delta_k)k & \text{if} \quad z = s \end{cases}, \tag{2.2}$$

where k' is capital installed in t to be ready to use in $t+1$, which depreciates at rate δ_k, and x is investment. Entrepreneurs have access to a CRS production function:

$$y = k^{\alpha}\ell^{1-\alpha}H^{\varphi}, \quad 0 < \alpha < 1, \quad \varphi \geq 0, \tag{2.3}$$

where ℓ is labor demand, H is the aggregate stock of human capital and φ captures the *external* effect of human capital.[7]

Upon observation of z, each entrepreneur decides how much to invest—if possible—how much of labor services to hire and how much to consume. Profits for a given stock of physical capital are given by output (2.3) minus payments to the labor factor $w\ell$, where w is the wage rate. A great simplification in the method of solution ensues by first working out labor hiring decisions by entrepreneurs. The following are results derived in straightforward fashion, working out the firm's optimization problem:[8]

$$\max_{\ell}[y - w\ell] = rk, \quad \ell = \frac{1-\alpha}{\alpha}\frac{r}{w}k, \quad y = \frac{r}{\alpha}k. \tag{2.4}$$

Notice that the profit function, labor demand and output supply are all linear functions of the capital stock in the hands of the entrepreneur. r is the entrepreneur's profit per unit of capital,

[7] I allow for the existence of an external effect of human capital, mainly because it helps with the calibration. Endogenous growth may arise even when $\varphi = 0$.

[8] A similar procedure was used in the literature before, for example, Angeletos [2007], Moll [2014] and Kiyotaki and Moore [2019]. By first working out labor hiring decisions, entrepreneurs profits and policy functions in general are linear in the relevant asset, and hence, aggregation is simple to carry out in spite of the underlying heterogeneity.

defined as:

$$r \equiv \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \left(\frac{H^{\varphi}}{w}\right)^{\frac{1}{\alpha}} w, \tag{2.5}$$

which is increasing in the aggregate stock of human capital—if there are external effects—and decreasing in the wage rate.

The results in (2.4) are useful because capital itself is not traded, but entrepreneurs can issue financial claims over capital that can circulate in the economy. Claims on capital can be issued over existing units of capital as well as over new units of capital. Let q be the price of claims over capital and n be the net claims over capital an entrepreneur holds. An entrepreneur's budget constraint is:

$$c_e = \begin{cases} [r + q(1-\delta_k)]\, n + (q-1)x - qn' & \text{if} \quad z = i \\ [r + q(1-\delta_k)]\, n + - qn' & \text{if} \quad z = s \end{cases}. \tag{2.6}$$

Appendix A.1 explains in more detail the financial market structure and these constraints. Here, I explain succinctly. The first term in the RHS of (2.6) shows net income from capital. The depreciation rate is shown explicitly since n are claims over capital. Expenditure in claims over capital is the last term in the RHS of the same equation. Note that in case the entrepreneur is an investor, he or she also has $(q-1)x \geq 0$ as income. When $q > 1$, an investor gains by producing new capital and sells claims at a higher price than the cost of production.[9]

Financial frictions are imposed as constraints on the fraction of capital that can be financed in the financial market. If the entrepreneur does not have an investment opportunity, he or she can sell claims up to ϕ over his or her existing acquired capital; I will incorporate a liquidity crunch by assuming that this parameter unexpectedly decreases and then recovers in a predictable fashion. If the entrepreneur has an investment opportunity, he or she can also sell claims up to

[9] If q were to be lower than unity, the entrepreneur can always abstain to produce new units of capital at no cost.

θ over new units of capital created; this parameter will be assumed to be constant.[10] Therefore, financial constraints are given by:

$$n' \geq \begin{cases} (1-\theta)x + (1-\phi)(1-\delta_k)n & \text{if} \quad z = i \\ (1-\phi)(1-\delta_k)n & \text{if} \quad z = s \end{cases}. \tag{2.7}$$

Constraint (2.7) says that claims over capital have to be at least $(1-\theta)$ of investment when having the opportunity. That is, an agent cannot issue claims over the entire new units of capital created. A higher θ means that credit will flow easily and that equity can be issued in a large fraction over new capital created. When having the investment opportunity, the agents can also sell claims over existing units of capital up to a fraction ϕ; therefore, this parameter measures the liquidity of financial markets. When being only savers, they are also affected by this liquidity constraint; however, since they cannot produce new capital, the credit constraint is not relevant for them.[11]

Let us now turn to workers.

Workers

There is a measure one of workers who aim to maximize:

$$\sum_{s=0}^{\infty} \beta^s u(c_{w,t+s}), \quad 0 < \beta < 1, \quad u(c_{w,t}) = \frac{c_{w,t}^{1-\sigma}}{1-\sigma}, \tag{2.8}$$

where $c_{w,t}$ is a worker's current consumption of the good, β is the discount factor, and σ is his

[10] Much of the literature treat this parameter as fixed, but, in principle, the troubles of the financial system can show up in both notions of financial availability. For simplicity, I follow the literature and assume θ constant throughout.

[11] Of course, entrepreneurs can always hold more claims than what is required from the financial constraint. That is, the constraints may be satisfied with strict inequality. Therefore, entrepreneurs can not only self claim all units of capital created but may also purchase more equity in the market beyond that point.

or her risk aversion. The period utility function $u(\cdot)$ and the discount factor β are the same as for entrepreneurs. Therefore, being able to create capital and output—that is, being an entrepreneur—is assumed not related to underlying preferences for consumption over time or differences in risk aversion.

As in Lucas [1988], I assume that they can devote u of their time to work for entrepreneurs. Let h be their stock of human capital; labor supply is given by $\ell^s = uh$. Labor is perfectly mobile, and hence, all entrepreneurs pay the same wage w. A worker's human capital is accumulated according to:

$$h' = [1 - \delta_h + \kappa(1-u)]h, \tag{2.9}$$

with the time endowment normalized to unity. δ_h is the depreciation rate of human capital, and κ measures the efficiency in the transformation of time to human capital accumulation. Finally, a worker's consumption is simply:

$$c_w = w\ell^s = wuh. \tag{2.10}$$

Notation for aggregate quantities

To display the definition of equilibrium in the next section, let me define some aggregate quantities in terms of notation. N denotes the aggregate demand of claims by entrepreneurs in the economy. H and K denote aggregate stocks of human and physical capital, respectively. C_e denotes the aggregate entrepreneur's consumption and C_w the aggregate consumption for workers. L denotes the aggregate demand of labor and L^s the aggregate supply of labor, while the aggregate output is denoted by Y.

2.1 Definition of equilibrium

Definition *A competitive equilibrium is a sequence of prices* $\{q_t, w_t\}_{t=0}^{\infty}$*, such that:*

1. *Entrepreneurs' policy functions* $c_{e,z}$*,* n'_z*, and* x *maximize their utility* (2.1) *subject to constraints* (2.2)*,* (2.6)*, and* (2.7)*.*

2. *Workers' policy functions* c_w, u, h' *maximize their utility* (2.8) *subject to the constraints* (2.9) *and* (2.10)*.*

3. *Claims on the capital market clear:*
$$N = K \tag{2.11a}$$

4. *Labor demand equals labor supply:*
$$L = L^s \tag{2.11b}$$

5. *Goods market clear:*
$$C_e + C_w + K' - (1-\delta)K = Y \tag{2.11c}$$

2.2 Solving the Model

Entrepreneurs

Let me start by using a guess-and-verify method and assume that in equilibrium:

$$q > 1. \tag{2.12}$$

Because it is so profitable to produce capital, investors will bind their financial constraint. Their budget constraint can therefore be written as:

$$c_e = \begin{cases} [r + \varrho(1-\delta_k)]\, n - \psi n', \;\; n' \geq 0 & \text{if} \quad z = i \\ [r + q(1-\delta_k)]\, n - q n', \;\; n' \geq (1-\phi)(1-\delta_k) n & \text{if} \quad z = s \end{cases}, \tag{2.13}$$

where:

$$\psi \equiv \frac{1 - q\theta}{1-\theta}, \quad \varrho \equiv \phi q + (1-\phi)\psi. \tag{2.14}$$

ψ is defined as the effective price of equity for investors, and $1 - q\theta$ is the down payment of investment. Since the investor self claims only $1-\theta$ of new units of capital created, the cost of one *unit* of equity is the down payment divided by $1-\theta$. For ψ to be well defined, it must be the case that $q > 1$, which is satisfied by assumption. Savers can sell at price q their current net holdings of equity, obtaining $q(1-\delta)n$ in the first term in brackets. Investors, since they are constrained, will value the liquid part ϕ of these units of claims at price q, and the complement $1-\phi$ will be valued at ψ; therefore, ϱ in (2.14) is defined as the effective resell price of equity. Note also that r only depends on economy-wide variables, not idiosyncratic variables. Hence, any individual amount of claims n chosen in the previous period delivers the same gain r, although the total profit rn will vary across entrepreneurs according to their holdings of n.

Appendix A.2 shows how to solve the entrepreneur's problem in such a way that the inherent heterogeneity does not prevent performing aggregation later in a very simple fashion. I present here the policy functions for entrepreneurs that depend on their current status:

$$\begin{gathered} \psi n_i' = \zeta_i \left\{ [r + \varrho(1-\delta_k)] n \right\}, \quad c_{e,i} = (1-\zeta_i) \left\{ [r + \varrho(1-\delta_k)] n \right\}, \\ x = \zeta_i \frac{[r + \varrho(1-\delta_k)] n}{1 - q\theta} - \frac{(1-\phi)(1-\delta_k) n}{1-\theta}, \end{gathered} \tag{2.15}$$

and:

$$qn'_s = \zeta_s \left\{[r + q(1-\delta_k)]n\right\}, \quad c_{e,s} = (1-\zeta_s)\left\{[r + q(1-\delta_k)]n\right\}, \tag{2.16}$$

ζ_i and ζ_s are the savings rates for investors and savers, respectively, defined as the fraction of wealth—not income—that is saved. These are chosen optimally to satisfy the following recursive equations:

$$(1-\zeta_i)^{-1} = 1 + \left\{\beta\left[\pi(1-\zeta'_i)^{-\sigma}\left(R'_{ii}\right)^{1-\sigma} + (1-\pi)(1-\zeta'_s)^{-\sigma}\left(R'_{is}\right)^{1-\sigma}\right]\right\}^{\frac{1}{\sigma}}, \tag{2.17}$$

$$(1-\zeta_s)^{-1} = 1 + \left\{\beta\left[\pi(1-\zeta'_i)^{-\sigma}\left(R'_{si}\right)^{1-\sigma} + (1-\pi)(1-\zeta'_s)^{-\sigma}\left(R'_{ss}\right)^{1-\sigma}\right]\right\}^{\frac{1}{\sigma}}. \tag{2.18}$$

Resources that an investor has at the beginning of the period are $[r + \varrho(1-\delta_k)]n$. Out of these resources, the investor decides to put a fraction ζ_i in equity expenditure $\psi n'_i$; the rest is consumed. How much to save depends on the anticipation of events that influences returns to savings that have idiosyncratic components. A similar characterization emerges for a saver.[12] Returns for the different assets have been defined as:

$$R'_{ii} \equiv \frac{r' + \varrho'(1-\delta_k)}{\psi}, R'_{is} \equiv \frac{r' + q'(1-\delta_k)}{\psi}, R'_{si} \equiv \frac{r' + \varrho'(1-\delta_k)}{q}, R'_{ss} \equiv \frac{r' + q'(1-\delta_k)}{q}. \tag{2.19}$$

Workers

Let us now turn to the workers' problem. The only decision for workers is to choose the amount of time devoted to work and to accumulate human capital (HK). The Bellman equation for

[12]When $\sigma = 1$, (the log utility case) it is straightforward to obtain $\zeta_i = \zeta_s = \beta$; this is called the "myopic" solution in in the portfolio literature, in which agents do not consider the expectation of how returns will evolve in the future to allocate their wealth. We will see that the value of σ will be important for the question at hand, and hence, assuming log utility is not innocuous.

workers with value function $\mathcal{W}(\cdot)$ can be stated as:

$$\mathcal{W}(h) = \max_{c_w} \left[u(c_w) + \beta \mathcal{W}'(h') \right], \tag{2.20}$$

subject to:

$$c_w + \frac{w}{\kappa}[h' - (1-\delta_h)h] = wh, \quad (1-\delta_h+\kappa)h \geq h' \geq (1-\delta_h)h. \tag{2.21}$$

The first equation in (2.21) has the following interpretation. The RHS is full income, which is income the worker would obtain if he or she devotes the entire unit of time working. In the LHS, we have consumption plus HK expenditures. Each unit of increase in HK has an effective cost of w/κ because forgone wages are adjusted by the efficiency in acquiring HK. A given increase in HK is less costly if attained by devoting less time to its acquisition. The inequality restrictions in (2.21) are simply $u \in [0, 1]$. In the numerical exercises of the paper, I will make sure that these are satisfied.[13]

I use again the same approach for solving the workers' problem as in the entrepreneur's problem. In this case, however, the process is simplified because there is homogeneity among workers; therefore, for the sake of brevity of exposition, the details will be omitted. Policy functions for workers are:

$$c_w = uwh, \quad u = (1-\zeta_w)\frac{1-\delta_h+\kappa}{\kappa}, \quad h' = \zeta_w(1-\delta_h+\kappa)h. \tag{2.22}$$

ζ_w is therefore the key variable for workers. It will be called "savings rate" as for entrepreneurs,

[13] u would never be chosen optimally to be 0 because then consumption would be zero. Inada conditions—satisfied for the utility function—prevent this from happening.

although savings here would be in terms of HK accumulation.[14] The higher ζ_w is, the more HK is acquired with less time working and less consumption. ζ_w satisfies the recursive equation:

$$(1-\zeta_w)^{-1} = 1 + \left[\beta(1-\zeta_w')^{-\sigma}(R_h')^{1-\sigma}\right]^{\frac{1}{\sigma}}, \tag{2.23}$$

and R_h' is defined as the return on HK accumulation:

$$R_h' = (1-\delta_h+\kappa)\Gamma_w', \quad \Gamma_w' = \frac{w'}{w}. \tag{2.24}$$

Hence, return to HK for workers is driven by the forgone wage to acquire HK versus the wage they expect to obtain in the future.[15]

The model cannot be solved unless it is normalized appropriately since there is perpetual growth. Appendix A.3 shows how to perform such normalization. Here, I state two important equations that are part of the system. The first equation can be obtained by labor market clearing. Aggregating over entrepreneurs and workers, equilibrium in the labor market is:

$$L \equiv \frac{1-\alpha}{\alpha}\frac{r}{w}K = uH \equiv L^s. \tag{2.25}$$

The second equation is the definition of r in (2.5). Both equations can be combined for two successive periods to obtain:

$$\frac{r'}{r}\Gamma_K = \Gamma_w'\frac{u'}{u}\Gamma_H, \quad \frac{r'}{r} = \left(\frac{\Gamma_H^{\varphi}}{\Gamma_w'}\right)^{\frac{1}{\alpha}}, \tag{2.26}$$

[14]Because the value of ζ_w determines u, ζ_w needs to satisfy the restriction

$$1 \geq \zeta_w \geq \frac{1-\delta_h}{1-\delta_h+\kappa},$$

, which is the restriction $u \in [0,1]$.

[15]Returns to HK accumulation will also be high the higher κ, the efficiency of transformation of time to HK, is and will be low the higher the depreciation rate δ_h is.

where $\Gamma_K = K'/K, \Gamma'_w = w'/w$ and $\Gamma_H = H'/H$ are the rates of growth of physical capital, wages and human capital, respectively.

Before going to the analysis of the dynamics, I point out a result concerning long-run growth.

2.3 Long-run behavior of the model: irrelevance of financial considerations

Let me focus on a balanced growth path (BGP), which amounts to analyzing the stationary state of the normalized system described in Appendix A.3. I am focusing on a BGP where r and u are time-invariant. As seen in the second equation in (2.26), an invariant r implies that wages and HK should be growing in a balanced way, and the first equation and the invariance of u imply that such behavior should also balance with the growth in physical capital. Denoting with bars the steady-state variables, equations (2.26) are written as:

$$1 = \left(\frac{\bar{\Gamma}_H^{\varphi}}{\bar{\Gamma}_w}\right)^{\frac{1}{\alpha}} \bar{\Gamma}_w, \quad \bar{\Gamma}_K = \bar{\Gamma}_w \bar{\Gamma}_H. \tag{2.27a}$$

HK growth in the steady state, from the last equation in (2.22) and savings rate from (2.23), is:

$$\bar{\Gamma}_H = \bar{\zeta}_w(1 - \delta_h + \kappa), \; (1 - \bar{\zeta}_w)^{-1} = 1 + \left[\beta(1 - \bar{\zeta}_w)^{-\sigma} \bar{R}_h^{1-\sigma}\right]^{\frac{1}{\sigma}}, \tag{2.27b}$$

where, from (2.24):

$$\bar{R}_h = (1 - \delta_h + \kappa)\bar{\Gamma}_w. \tag{2.27c}$$

A partial system for the economy is obtained by using the second equation in (2.22):

$$\bar{u} = (1 - \bar{\zeta})\frac{1 - \delta_h + \kappa}{\kappa}. \tag{2.27d}$$

System (2.27) can be solved, provided values for the parameters, for all BGP growth rates of both types of capital, wages and time devoted to work. Note that the rate of growth of output $\Gamma_Y = Y'/Y$ from the aggregate version of the last equation in (2.4), is $\bar{\Gamma}_Y = \bar{\Gamma}_K$ in the steady state. Hence, the first relevant result delivered by this model is that the long-run *rate of growth* of the economy is unaffected by any financial conditions of the economy, and not θ or ϕ or even π appear in these equations.[16]

I now turn to a calibration of the model in order to then focus on the implications of a transitory liquidity crunch.

2.4 Calibration

The objective of the numerical exercises of the paper is not the give exact quantitative predictions of a liquidity crunch. However, even the qualitative answers may depend on the parametrization of the model; therefore, I provide in this section a rough calibration to build confidence about the results of the credit crunch. I will consider a quarterly economy.

I take into account an annual rate of growth of per capita GDP of 2% as a long-run measure of growth. Lucas [1988] argue that an annual rate of growth for HK is 0.9%. Therefore, I set $\bar{\Gamma}_K = 1.00496$ and $\bar{\Gamma}_H = 1.00224$ for both types of capital. I set $\alpha = 0.36$, as is standard in the

[16]In part, this result hinges on the assumption that workers do not participate in the asset market, but this result is nevertheless not obvious as there are general equilibrium effects of financial factors that could have been manifested, for example, in the wage growth, and impacted this system and hence the long-run growth.

literature, and use (2.27a) to determine the value of φ. Using the second equation in (2.27a), I obtain $\bar{\Gamma}_w = 1.0017062$.[17] From the first equation in (2.27a), it is possible to find $\varphi = 0.776$, which measures the strength of the spillover of HK in production.[18]

I also set $\beta = 0.985$, consistent with most values used in the literature for a quarterly economy. It turns out that the actual value of σ is important for the results on the persistent effect of a credit crunch on the trend of GDP. Hence, we will look at different values for σ in the exercises below. As a benchmark value, let me set $\sigma = 2$ for now, and then, from the second equation in (2.27b), I obtain $\bar{R}_h = \bar{\Gamma}_K^{\sigma}/\beta = 1.025$, a 10% of return to HK accumulation.[19] To find the value of $\bar{\zeta}_w$, we can work with the first equation in (2.27b) and (2.27c) to obtain $\bar{\zeta}_w = \bar{\Gamma}_K/\bar{R}_h = 0.98$. Next, I impose $\bar{u} = 0.7$, which implies a 70% of time devoted to work and 30% of time devoted to HK accumulation. Then, using (2.27d) and (2.27c), I obtain $\kappa = (1-\bar{\zeta})/u \times \bar{R}_h/\bar{\Gamma}_w = 0.029$. Finally, we can use equation (2.27c) again to obtain $\delta_h = 1 + \kappa - \bar{R}_h/\bar{\Gamma}_w = 0.0064$, so there is an annual depreciation of HK of 2.5%.

Let us now go into the entrepreneurs parameters. Key parameters are the financial friction parameters θ, ϕ and the probability of finding investment projects π. I follow related literature, such as Kiyotaki and Moore [2005], Shi [2015], Nezafat and Slavick [2015] and Del Negro et al. [2017], and set $\theta = \phi = 0.25$ and $\pi = 0.05$.[20] I use the standard value for the depreciation rate of $\delta_k = 0.025$.

With this calibration, an equilibrium for the key variables of the economy in the steady state

[17] This corresponds to a roughly 1% increase in real wages per year. In this model, whenever the rate of growth of physical capital differs from that of HK, wages will be growing.

[18] This means that if human capital increases 1%, output increases through human capital spillover alone by 0.77%.

[19] Despite the very broad definition of HK implicit in this model, this value appears to lie within the set of values considered to be measuring returns to human capital in practice; see, for example, Carneiro et al. [2011].

[20] Most of the qualitative results of the paper are maintained if we take alternative parametrizations. Alternative parametrizations must, notwithstanding, yield an equilibrium value for $\bar{q} > 1$. For this to happen, θ, ϕ and π cannot be too large.

can be found; I omit these results due to space limitations and the focus of the paper. It is instructive, however, to make explicit the different ex post returns on equity in the steady state. These are given—for the benchmark case of $\sigma = 2$—by:

$$\bar{R}_{si} = 0.716 < \bar{R}_{ss} = 1.014 < \frac{1}{\beta} = 1.015 < \bar{R}_{ii} = 1.21 < \bar{R}_{is} = 1.71. \tag{2.28}$$

Because of endogenous growth, the capital stock is perpetually growing, and therefore, it is convenient to think about the entrepreneur's portfolio decisions in terms of normalized equity, namely, equity divided by the capital stock. Returns in (2.28) portray how dynamics are still present in the steady state. Returns for investors are always higher than the discount rate, inducing them to acquire more normalized equity. Savers face a return inferior to the discount rate, therefore decreasing their normalized equity holdings. However, since status is i.i.d., in each period, a fraction π will be acquiring more equity and a fraction $1 - \pi$ will be reducing their equity in normalized terms. It is this configuration of behavior that balances in such a way as to maintain aggregate normalized equity constant.[21]

3 The effects of a transitory liquidity crunch

The economy is assumed to be in steady state, and unexpectedly, the parameter ϕ drops. After this surprise change, it recovers gradually and predictably. This is what I call a *liquidity crunch* in the model and is meant to capture—albeit in a crude manner—the behavior of the financial markets in the Great Recession. The model is solved for risk aversion values of $\sigma =$

[21] Savers purchase normalized equity from investors continuously in the steady state, but the ex post returns for them are always inferior to the discount rate. This situation happens because for savers, the next period amount of normalized equity is lower than the current value, whereas it is higher than the un-depreciated amount. Therefore, the savers purchase normalized equity from investors at just slightly more than needed to cover depreciation, but not enough to maintain or increase their normalized equity holdings that they had in the previous period.

$\{0.9, 1, 2, 4, 7, 12\}$.[22] Figure 2a shows how liquidity is completely frozen at impact and then recovers; naturally, its behavior does not depend on σ.

Figure 2b portray perhaps the most important result of the paper. For $\sigma = 0.9$, there is a sharp downward deviation from the trend but later a strong recovery in such a way as to put the economy on a *higher* trend than before the crunch. For $\sigma = 1$, the behavior is similar, but GDP returns again to its previous trend. For values of σ equal to 7 and 12, GDP never falls below the trend, and it actually converges to a higher trend than before the crunch. It is for moderate values of σ, equal to 2 and 4, that we observe a downward deviation with GDP never returning to its previous trend. For these values of σ, the economy never grows back strongly enough to attain the previous trend: the *transitory* credit crunch produces a *permanent* deviation in the trend. To interpret and gain intuition about the paths of the variables in figure 2, it is convenient to examine the market for equity because equilibrium in this market determines the capital stock and influences labor demand, which in turn influence workers' HK decisions. Appendix A.4 shows that normalized *demand* of equity is given by:[23]

$$D = \underbrace{\zeta_s \frac{r + q(1-\delta_k)}{q}(1-\pi) - (1-\delta_k)(1-\pi)\,.}_{\text{Normalized supply of funds}} \tag{3.1a}$$

Furthermore, the normalized *supply* of equity is:

$$S = \underbrace{\theta \left\{ \zeta_i \left[\frac{r}{1-q\theta} + \left(\frac{\phi q}{1-q\theta} + \frac{1-\phi}{1-\theta} \right)(1-\delta_k) \right] - \frac{(1-\phi)(1-\delta_k)}{1-\theta} \right\} \pi + \phi(1-\delta_k)\pi\,.}_{\text{Normalized demand of funds}} \tag{3.1b}$$

[22] These values are chosen because they encompass the most common in the literature, although values of σ lower than one and higher than 6 are usually considered unrealistic. I use Dynare to solve the model, and I report the impulse response functions. The model could also have been solved with a deterministic simulation given the time path of ϕ, which will give the same results.

[23] Normalized demand and normalized supply in equations (3.1) refer to demand and supply of equity divided by the capital stock.

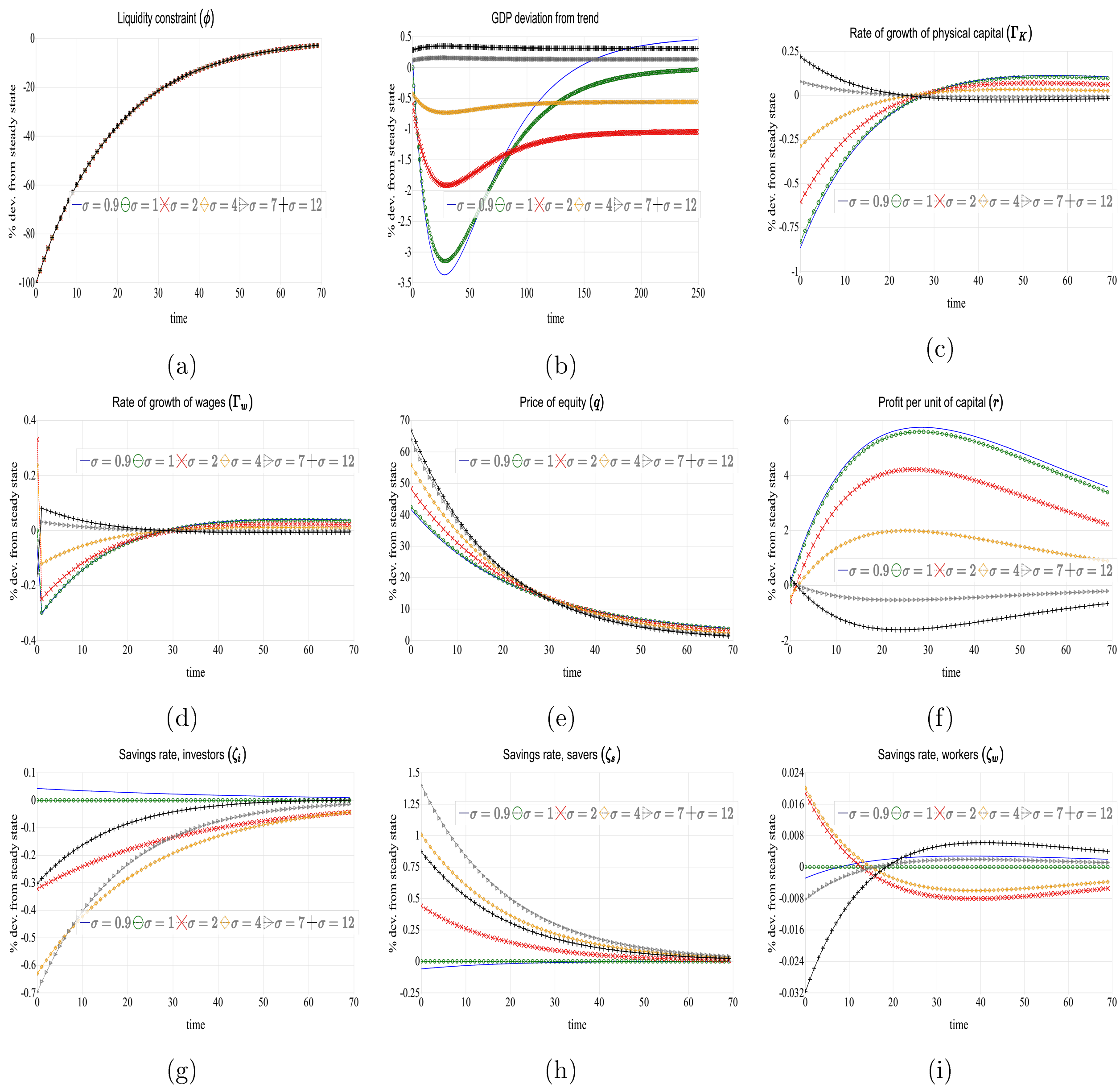


Figure 2: Response over time with the liquidity crunch.

Demand of equity is the supply of funds, and the supply of equity is the demand of funds. From a partial equilibrium perspective, we can view both D and S as functions of q, and there must be a q equating D and S. Equilibrium—$D = S$—should be attained of course at all times, but later, I will focus on movements of these curves and the resulting equilibrium normalized equity to say something about physical capital growth. For example, imagine that $D = S$ in the steady state with a constant value of normalized equity. This constant value means that both equity and capital (over which equity is issued) are growing at a common rate. Now, if there is a surprise increase in D, for example, then the equilibrium normalized equity should rise (along with q), but this means that the rate of growth of equity and capital must increase compared to the steady state. This insight will be used to understand changes in liquidity.

In the discussion of the previous paragraph, I assumed that D is a downward sloping function of q while S is an upward sloping function of q. This is certainly the case—and can be verified directly—for log utility ($\sigma = 1$) because $\zeta_i = \zeta_s = \beta$ and equations in (3.1) do not involve future values. Policy functions for workers (2.22) adopt simple expressions as well since $\zeta_w = \beta$.[24] However, nonmyopic behavior—when $\sigma \neq 1$—has important implications in the paper, so it is better to obtain a sharper characterization of the savings rates. To this end, I linearize the equations of motion for these variables, equations (2.17), (2.18) and (2.23). Appendix A.5 shows the details.

Defining:

$$x_{i,t} \equiv (1 - \zeta_{i,t})^{-1}, \quad x_{s,t} \equiv (1 - \zeta_{s,t})^{-1}, \quad x_{w,t} \equiv (1 - \zeta_{w,t})^{-1}, \tag{3.2}$$

as the inverse of one minus the savings rates for entrepreneurs and workers, respectively. Lin-

[24] An important result under workers' myopic behavior is then that HK growth and time working are completely unaffected with the liquidity crunch.

earizing first the system (2.17) and (2.18) around the steady state:

$$\hat{\mathbf{x}}_t = (1-\sigma)\sum_{s=1}^{\infty}\beta^s \mathbf{B}^{s-1}\mathbf{A}\hat{\mathbf{R}}_{t+s}, \tag{3.3}$$

where hats over variables denote deviations from the steady state, $\mathbf{A}$ and $\mathbf{B}$ are matrices of dimensions 2×4 and 2×2 of positive elements, respectively, and $\hat{\mathbf{x}}_t = [\hat{x}_{i,t}, \hat{x}_{s,t}]'$ and $\hat{\mathbf{R}}_t = [\hat{R}_{ii,t}, \hat{R}_{is,t}, \hat{R}_{si,t}, \hat{R}_{ss,t}]'$.

Entrepreneurs' savings rates depend on the present discounted value (PDV) of returns. This expected value includes the likelihood of changing status—between being an investor or saver—through time. While σ appears in matrices $\mathbf{A}$ and $\mathbf{B}$ (see equations (A.35) and (A.36) in Appendix A.5), the sign of the PDV of returns does not depend on its actual value, and hence, whether σ is above or below one will have opposite effects on the savings rates.

Similarly, linearizing the equation of motion for the worker's savings rate of (2.23), using the definition in (3.2), I obtain:

$$\hat{x}_{w,t} = (1-\sigma)\sum_{s=1}^{\infty}\beta^s b^{s-1} a \hat{R}_{h,t+s}, \tag{3.4}$$

where a and b are positive constants. Therefore, the savings rate for workers depends on the PDV of HK returns, and as for entrepreneurs, whether σ is above or below one will have opposite effects on the savings rate and therefore opposite effects on HK accumulation.

We want to understand the impact of a surprise downward change in ϕ and predictable recovery of the type portrayed in figure 2a. I will divide the analysis according to the value of σ.

The case $\sigma = 1$

Let me start with the simplest "myopic" case of $\sigma = 1$.[25] Let us examine first the immediate change in the equity market when ϕ drops. S in (3.1b) clearly decreases with ϕ, which can be seen by direct differentiation of (3.1b) and the fact $q > 1$. Note also that D in (3.1a) does not change: equilibrium then requires a rise in q. However, there are of course general equilibrium effects because equity and capital growth decrease; this pushes wage growth down as HK growth is not altered. Therefore, r increases, and equations (3.1) show that D will increase and S will shift back. Ultimately, however, we know that equity growth must fall below the steady state along with capital growth and wages; this is seen in figures 2c and 2d. This behavior lasts for some periods after reverting patterns. To simplify the discussion, I will call these two different phases as "short run" and "long run", respectively. I will also use the terms "low" and "high" instead of below steady state and above steady state. Therefore, in the short run, wage growth is low, but it is high in the long run, likewise for the growth of capital. Why do capital and wage growth have to be high in the long run? The reason is that r must be the same in the steady state as that prior to the crunch by definition of the BGP. Looking at the definition of r in (2.5), it is clear that low wage growth in the short run makes r high as HK keeps growing at the same rate. When wage growth crosses zero from below in figure 2d, the level of r is the highest above the steady state, as seen in figure 2.5. For r to return to the steady state, wage growth must be high for a long time. The fact that r is high means that both D and S increase, making the growth of equity and capital high in the long run, as we see in figure 2c. Workers being myopic do not change their HK accumulation pattern. Physical capital growth then, being low in the short run and high in the long run, produces compensating effects on GDP so that it reverts to its previous trend.

The case $\sigma = 0.9$

[25]In which case the savings rates are time-invariant and equal to β. The last row of figure 2 show the behavior of the three savings rates, figures 2g, 2h and 2i, which are of course unchanged from the steady state.

Take now the case $\sigma < 1$. In this case, expectations matter for savings rates. As shown in figure 2b, the contraction is sharper in the short run, but so is the recovery, and GDP ends up in a higher trend. Note that the behavior of the growth of capital, wages, the price of capital and r are very similar to the case $\sigma = 1$. Therefore, why is such a divergence for the trend observed? We know that ϱ is negatively affected by the decrease in ϕ and that this negatively impacts returns conditional on being an investor; see R_{ii} and R_{is} in (2.19). We know then, from (3.3), for a fact, that both ζ_i and ζ_s would decrease if the path of ϕ would be the only change; this would induce both curves D and S in (3.1) to decrease further compared to the $\sigma = 1$ case. Moreover, this effect would reduce capital growth even further.[26] Considering the general equilibrium effects, especially through the rise in r, the equilibrium value of ζ_i actually increases along the adjustment, as seen in figure 2g. Note that the higher detrimental effect in the short run for growth of capital must be compensated with higher growth in the long run, again to maintain r at the same level in the BGP. Now, for workers, we can see that the effect on ζ_w derives entirely through the general equilibrium effect on wages. In the short run, the PDV of HK returns is negative because of the drop in wage growth caused by reduced demand of labor, and then the savings rate is low, implying lower HK accumulation and more time devoted to work. The substitution effect (SE) here dominates for ζ_w, as future consumption becomes more expensive and more time is devoted to work at the expense of HK accumulation. In the long run, workers expect high wage growth, and therefore, PDV is positive, SE dominating implies that ζ_w is high, and HK investments increase, while time devoted to work decreases. Note the important result that in the long run, both physical and HK are growing above the steady state. This finding, in turn, implies that the GDP trend ends up being above the trend prior to the crunch.

[26]This can be interpreted as the substitution effect (SE) dominating over the income effect (IE) for savings rates, which can be most clearly seen in the policy functions for consumption in (2.15) and (2.16). A reduction in ζ_i and ζ_s increases consumption. Then, when ϕ drops, the PDV of returns becomes negative, and equation (3.3) shows that savings rates must decrease. Because returns to savings have dropped, IE tends to reduce consumption, but the SE makes current consumption less expensive, and when $\sigma < 1$, this effect dominates.

The case $\sigma = 2, 4$

Let us take the cases of moderate—more empirically plausible—values for σ, such as 2 and 4 in the figures. In these cases, the new trend is below the previous trend, as seen in figure 2b. Note that as seen in figures 2c, 2d, 2e and 2f, the general pattern of the variables is similar to lower values for the risk aversion. It is again the behavior of the savings rates that influences the trend behavior. A reduced path of ϕ would make PDV of returns negative for entrepreneurs, but now, ζ_i and ζ_s would increase, as inferred from (3.3). Therefore, compared with the case $\sigma = 1$, D would shift to the right and S would not decrease by much. However, the decrease in S would dominate, and equity growth and capital growth decrease in the short run. Now, the equilibrium value of ζ_i is low in the short run due to all general equilibrium effects. Therefore, the situation is similar to that previously, with capital growth low in the short run along with wage growth and in the long run high growth in both capital and wages. For workers, low wage growth in the short run makes their PDV of HK returns negative and therefore their ζ_w high, accumulating more HK. In this case, the income effect (IE) dominates, reducing time devoted to work, and this can be seen in figure 2i. In the long run, workers foresee high wage growth and hence high returns to HK accumulation, making PDV of returns positive. Because IE dominates, ζ_w decreases, and hence, HK growth is low and more time is devoted to work. Because HK growth is low in the long run, there is a negative effect on productivity that dominates the high physical capital growth and the GDP trend ends up below the pre-crunch trend.

The case $\sigma = 7, 12$

Finally, I examine the case of higher risk aversion, where σ equals 7 and 12 in the figures. As we can see in figure 2b, there is actually a slight expansion in the short run, and the economy ends up in a higher trend in the long run. We know that PDV of returns for investors and savers is negative since the path of ϕ is lower than before; this induces a strong positive response in the savings rates ζ_i and ζ_s. Compared to the case of $\sigma = 1$, D increases strongly to the right, while S tends to increase with ζ_i but is negatively influenced by the drop in ϕ. The overall effect, including general equilibrium effects, as seen in figure 2c, is an increase in the rate of growth of capital. Note that the increase in D is strong as q increases strongly in the short run in figure 2e. Because growth of capital is high, demand of labor is also high with a corresponding high wage growth in the short run, as seen in figure 2d. Since wage growth is high in the short run, workers in this phase face positive PDV of HK returns, and therefore, the savings rate is low, with IE dominating. There is low HK growth and high time devoted to work in the short run. Afterward, however, in the long run, wage growth decreases below the steady state along with capital growth. This must occur again so that r would converge back to its level before the crunch so that BGP is satisfied. In this phase, because wage growth is low, PDV for workers is negative, and hence, ζ_w increases, with IE dominating. Workers accumulate more HK and work less, as seen in figure 2i. The implication is that because HK growth is high in the long run, the trend of GDP ends up at a higher level.

The main message of the analysis above is that the model is predicting a nonmonotone effect on the trend of GDP as risk aversion increases. For realistic moderate values of risk aversion—above one but not too high—the transitory credit crunch produces a downward permanent deviation in the GDP trend of the type that could have happened in the data as portrayed in figure 1 in the Introduction.

One salient feature of the results shown above is that the price of equity in all scenarios increases with the liquidity crunch. This situation caused major concern in the literature that studied liquidity fluctuations as introduced in this paper (see, for example, Shi [2015]). The concern is that the liquidity crunch would be implying an asset market boom rather than a contraction. In our application, this situation may also cause further concern since equilibrium returns for entrepreneurs are directly influenced by the price of equity as are the savings rates, which may change the conclusions of the study. In the next section, therefore, I extend the model by incorporating a storage technology that overturns the result of an increase in the price of equity at the moment of the crunch, and I examine the implications for the question at hand regarding the GDP trend.

3.1 An extension: Entrepreneurs with access to storage technology

I assume that storage is available to all entrepreneurs; we might think about this asset as a valued commodity or foreign currency. Entrepreneurs want to maximize (2.1) as before, but now, their budget constraint is:

$$c_e = \begin{cases} [r + \varrho(1-\delta_k)]\, n + m - \psi n' - pm', n' \geq 0, & m' \geq 0 \quad \text{if} \quad z = i \\ [r + q(1-\delta_k)]\, n + m - qn' - pm', n' \geq (1-\phi)(1-\delta_k)n, & m' \geq 0 \quad \text{if} \quad z = s \end{cases}, \tag{3.5}$$

where m is the storage good. To acquire m' units of this good for next period, they need to invest pm' out of current resources. For simplicity, I will set $p = 1$. Appendix B shows the details of this extension. Investors will not store any goods because of the gains of using equity when $q > 1$, which will still hold in equilibrium. Savers then need to divide their savings in equity and the storage good. Their policy functions are given by:

$$\begin{gathered} \psi n_i' = \zeta_i \left\{ [r + \varrho(1-\delta_k)]n + m \right\}, \quad c_{e,i} = (1-\zeta_i) \left\{ [r + \varrho(1-\delta_k)]n + m \right\}, \\ x = \zeta_i \frac{[r + \varrho(1-\delta_k)]n + m}{1 - q\theta} - \frac{(1-\phi)(1-\delta_k)n}{1-\theta}, \quad m' = 0, \end{gathered} \tag{3.6}$$

and:

$$qn'_s = \mu\zeta_s \left\{[r + q(1-\delta_k)]n + m\right\}, \quad c_{e,s} = (1-\zeta_s)\left\{[r + q(1-\delta_k)]n + m\right\},$$
$$pm' = (1-\mu)\zeta_s \left\{[r + q(1-\delta_k)]n + m\right\}. \tag{3.7}$$

ζ_i satisfies (2.17), the same equation as before. In addition, μ is the fraction of total savings put in equity, which satisfies the equation:

$$\pi(1-\zeta_i')^{-\sigma}[\mu R'_{si} + (1-\mu)R'_m]^{-\sigma}(R'_{si} - R'_m)(1-\pi)(1-\zeta_s')^{-\sigma}[\mu R'_{ss} + (1-\mu)R'_m]^{-\sigma}(R'_{ss} - R'_m) = 0, \tag{3.8}$$

where the return on storage good is simply $R'_m = 1/p = 1$. The savings rate ζ_s now satisfies:

$$(1-\zeta_s)^{-1} = 1 + (\beta\{\pi(1-\zeta_i')^{-\sigma}[\mu R'_{si} + (1-\mu)R'_m]^{1-\sigma} + (1-\pi)(1-\zeta_s')^{-\sigma}[\mu R'_{ss} + (1-\mu)R'_m]^{1-\sigma}\})^{\frac{1}{\sigma}}. \tag{3.9}$$

Appendix B.1 shows the details of solving the model. Here, I want to present the relationships among the returns on the different assets in the steady state for the benchmark case $\sigma = 2$:

$$\bar{R}_{si} = 0.744 < \bar{R}_m = 1 < \bar{R}_{ss} = 1.0157 < \frac{1}{\beta} = 1.0152 < \bar{R}_{ii} = 1.840 < \bar{R}_{is} = 1.616. \tag{3.10}$$

A similar configuration as before emerges where investors have high returns on equity and accumulate more normalized equity each time. When they become savers, they decrease their normalized equity holdings while accumulating the normalized storage good. The normalized storage good is dominated in return, but they decide a portfolio balance between equity and the storage good according to (3.8) considering future returns. They decide to invest in the storage good because they may become investors next period, and then, they can sell this good to finance investment.

Figure 3 the effects of a transitory liquidity crunch on the economy for the benchmark case of $\sigma = 2$. I compare the previous situation of no storage good with the case of the existence of a storage good. Figure 3a simply shows the same liquidity crunch as before, the exogenous downward shift in ϕ. Figure 3b shows that the short-run contraction is much more severe with the storage good, and the permanent downward deviation in trend is more pronounced as well. Figure 3c shows that the drop in the rate of capital growth is more pronounced than before. I also show now the rate of growth of GDP (Γ_Y in figure 3j) which also shows a deep contraction. Since entrepreneurs can store the good, a "flight to quality" decreases equity demand; this can be corroborated in figure 3k, which shows that the portfolio decision of savers now changes to less equity and more storage. Figure 3d shows again that wages are mostly influenced by demand of labor that is reduced in the short run by the drop in growth of capital. Note now that the rebound toward the long run in wage and capital growth is stronger than before. Figure 3e shows that now there is an asset price decrease at impact, although quantitatively small in this parametrization, but q barely increases later and throughout the transition. Therefore, this more realistic characterization of the liquidity crunch does not overturn the result that the transitory liquidity crunch produces a permanent trend break in GDP; in contrast, it magnifies the result.

3.2 On the plausibility of the mechanism

This paper explored an equilibrium approach to explain a phenomenon that arguably took place since the Great Recession. By this, I mean that I took a modeling approach where markets are in equilibrium—in particular, the labor market—and trace out the macroeconomic consequences of a liquidity crunch. It is true that unemployment was an important feature during the Great Recession, but the aim of the present study is to first understand if a flexible price, equilibrium

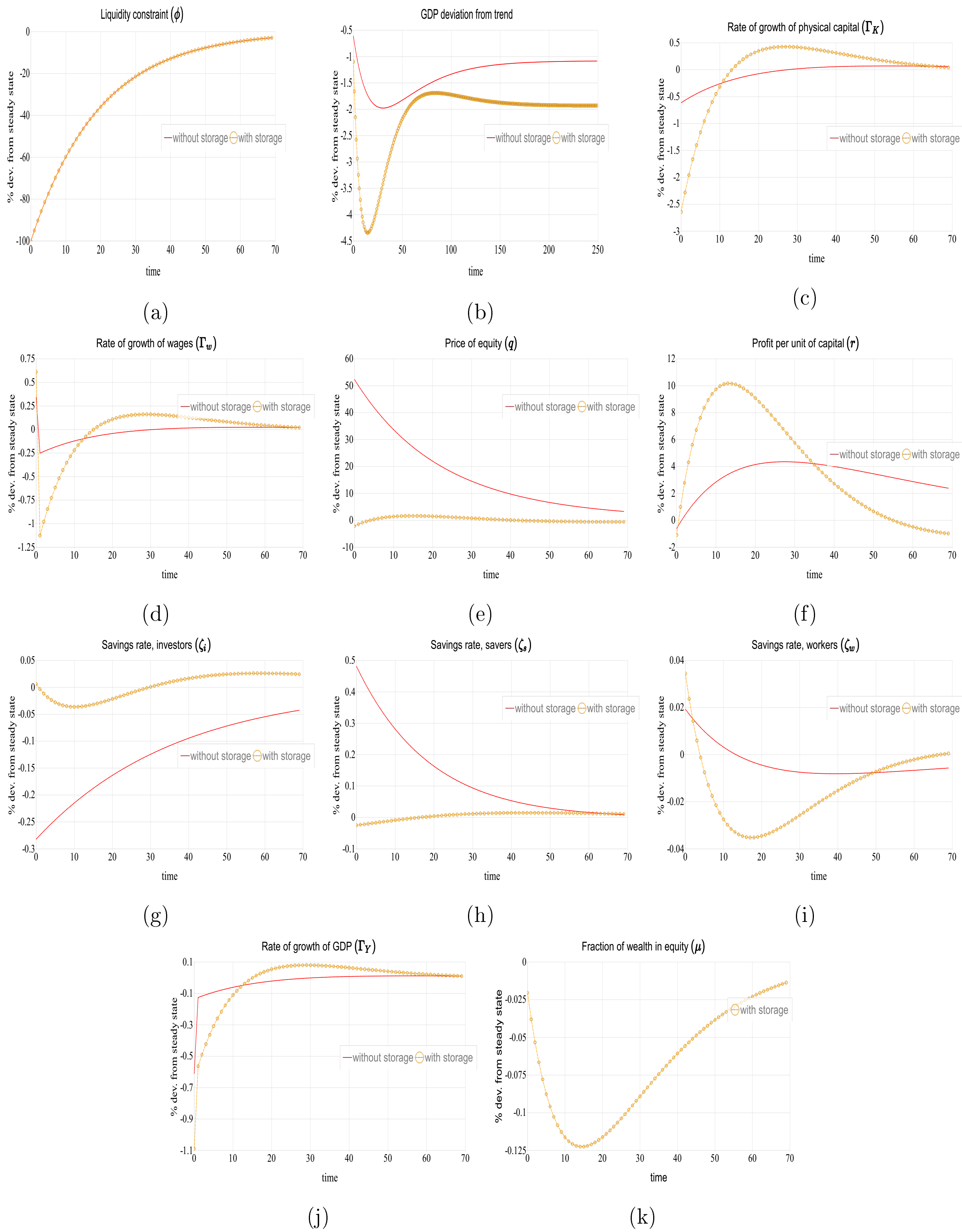
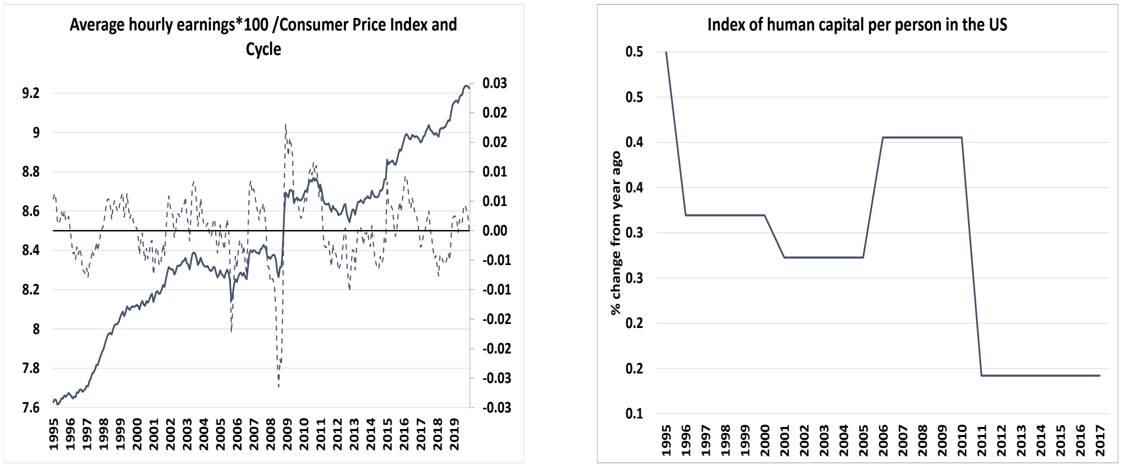


Figure 3: Response over time with the liquidity crunch and with storage good.



(a) Real wages and its cycle (right axis).

(b) Percent change in US human capital.

Figure 4: Wages and human capital.

model can shed light on understanding the phenomenon. That said, I am not claiming that this paper proposes a full or *the* explanation of the phenomenon; rather, proposed here is a plausible mechanism that might have played a role. In this regard, it is important to examine some of the testable implications of the model. Let me start with the behavior of wages near the Great Recession. Figure 4 shows measures of real wages in the US. In figure 4a, I show average hourly private earnings of production and nonsupervisory employees divided by the CPI and multiplied by 100 and the HP cycle that I extracted from the series (with $\lambda = 1600$). Since at least 1995, we can see an increase in this measure of real wages, which agrees with the model due to the presence of external effects of HK in the production function. Most importantly, we can see there is a sizable drop in wages in 2008 and later a strong recovery until 2011. Therefore, the general pattern of wages in the model agrees with this measure in the data. Of course this crude measure of wages might be too simplistic to argue that the mechanism of the model holds in the data, but at least, it does not preclude the possibility. Figure 4b shows a measure of HK for the US, where this series originated from the Penn World Table (PWT). The figure shows a decline in the growth rate of human capital, from approximately 0.4% annually in 2010 to approximately 0.14% annually from 2011 forward. For a further discussion of the most recent version of the PWT data, see Feenstra et al. [2015]. The PWT reports human capital accumulation as average years of schooling in the population and combines data from Barro and Lee [2013] and Cohen and Leker [2014].

Wage rigidity has been noted as a cause for acyclicality of real wages over the business cycle.[27] Some authors, however, studied in depth the labor market during the Great Recession and reached different conclusions. For example, Schaefer and Singleton [2017] use employer-employee panel data and show that UK firms were able to respond to the Great Recession with substantial real wage cuts and by recruiting more part-time workers. The authors also find that hours in entry-level jobs of new hires were also substantially procyclical. Moreover, Elsby et al. [2016], using 1979–2012 CPS data for the United States and 1975–2012 NES data for Great Britain, study wage behavior in both countries, with particular attention to the Great Recession. The authors conclude that real wages are procyclical in both countries.

4 Concluding remarks

I have developed a model where a transitory liquidity crunch produces a permanent break in the GDP trend. The model combines liquidity shocks of the type proposed by Kiyotaki and Moore [2019] with human capital as the engine of endogenous growth as in the seminal work of Lucas [1988]. The model provided a plausible mechanism to understand why despite the recovery of the growth of the economy after the Great Recession, it appears that the GDP trend is not the same as that prior to the crisis.

[27]See Solon et al. [1994] for a study that challenged the conventional view at the time that real wages were largely acyclical.

Appendix

A The basic model

A.1 Entrepreneurs feasibility sets

While each entrepreneur manage k to produce with technology (2.3), he or she does not necessarily get all profits from the activity. Entrepreneurs are allowed to issue claims on capital each period (subject to financial constraints), and acquire claims issued by some other agents. At the beginning of a period an entrepreneur will be managing an stock of capital k with an ownership structure determined by past decisions. Let e be the fraction of capital over which claims are currently issued and a be claims on capital issued by other entrepreneur. The beginning of period net claims or net worth is therefore $n = k - e + a$. The financial assets e and a are claims over an homogenous factor of production which is paid r, the profit per unit of capital defined in 2.5, and are subject to the same inter-period frictions:

$$a' - (1 - \delta_k)a \geq -\phi(1 - \delta_k)a, \quad 0 \leq \phi \leq 1, \quad z = i, s \tag{A.1a}$$

$$e' - (1 - \delta_k)e \leq \phi(1 - \delta_k)(k_i - e), \quad 0 \leq \phi \leq 1, \quad z = s. \tag{A.1b}$$

The first restriction, (A.1a) states that if agents want to sell claims on capital managed by others they can do so only until a fraction ϕ of previous period's claims and in (A.1b) it is assumed that new claims can be issued at most up to ϕ of previous period self-claimed capital $k - e$.[28]

[28]Both restrictions potentially prevent the agent from obtaining the desired amount of funds in the current period. Note that the depreciation of capital over which claims are issued is made explicit in the formulation of the frictions. This simplifies the exposition.

The reason why (A.1b) does not hold for a current investor is that he or she is able to create new units of capital x and may issue claims over these new units. It is assumed that he or she can do so up to a fraction θ of those new units created:

$$e' - (1-\delta_k)e \leq \phi(1-\delta_k)(k-e) + \theta x, \quad 0 \leq \theta \leq 1, \quad z = i. \tag{A.1c}$$

The budget constraints are:

$$c_e + q[a' - (1-\delta_k)a] = rn + q[e' - (1-\delta_k)e], \quad z = s \tag{A.2a}$$

$$c_e + x + q[a' - (1-\delta_k)a] = rn + q[e' - (1-\delta_k)e], \quad z = i. \tag{A.2b}$$

where q is the price of claims over capital.

A current investor has to decide on the amount of goods $x \geq 0$ to use into new capital creation according to (2.2).

Since both self-claimed capital $k - e$ and claims on capital managed by others a pay the same profit r, and they are both subject to the same liquidity friction, they are equivalent as a means of savings, the composition of n is undetermined. It is possible then to express the agents' problem in terms of a single financial state n: replacing the law of motion of capital, equation (2.2), in the investor's budget constraint, his constraint set can be simplified to:

$$c_e + x + q[n' - x - (1-\delta_k)n] = rn, \tag{A.3a}$$

with the finance constraint:

$$n' \geq (1-\theta)x + (1-\phi)(1-\delta_k)n. \tag{A.3b}$$

Similarly, for the saver:

$$c_e + q[n' - (1-\delta_k)n] = rn, \tag{A.3c}$$

with the finance constraint:

$$n' \geq (1-\phi)(1-\delta_k)n. \tag{A.3d}$$

The assumption of the model is that $q > 1$, if that is the case, (A.3b) is satisfied with equality as investors would like to make x as large as possible. Taking into account (A.3a) and (A.3b), we get the first equation in (2.6), and (A.3c) is the second equation in (2.6).

A.2 Solving entrepreneur's problem

To solve the model let me redefine the budget constraints in (2.13) in terms of the resources entrepreneurs have at the beginning of any period:

$$c_e = \begin{cases} \omega_i - \psi n', & \omega_i = [r + \varrho(1-\delta_k)]\, n \quad \text{if} \quad z = i \\ \omega_s - qn', & \omega_s = [r + q(1-\delta_k)]\, n \quad \text{if} \quad z = s \end{cases}. \tag{A.4}$$

Let $c_{e,z}$ and n'_z be the policy functions for investors and savers. Note that the evolution of wealth between two periods is given by:

$$\begin{aligned} \omega'_{ii} = [r' + \varrho'(1-\delta_k)]\, n'_i \quad \omega'_{is} = [r' + q'(1-\delta_k)]\, n'_i \quad \text{if} \quad z = i, \\ \omega'_{si} = [r' + \varrho'(1-\delta_k)]\, n'_s \quad \omega'_{ss} = [r' + q'(1-\delta_k)]\, n'_s \quad \text{if} \quad z = s. \end{aligned} \tag{A.5}$$

I assume that the value function depends on current wealth ω_z, all current prices in the economy, and its functional form inherits that of the utility function. The value function also depends on the status z. Therefore Bellman equations for entrepreneurs are:[29]

$$\mathcal{V}_i(\omega_i) = \max_{c_e,n'} \left\{ u(c_e) + \beta \mathbb{E} \left[\pi \mathcal{V}_i'(\omega_{ii}') + (1-\pi) \mathcal{V}_s'(\omega_{is}') \right] \right\}, \tag{A.6}$$

$$\mathcal{V}_s(\omega_s) = \max_{c_e,n'} \left\{ u(c_e) + \beta \mathbb{E} \left[\pi \mathcal{V}_i'(\omega_{si}') + (1-\pi) \mathcal{V}_s'(\omega_{ss}') \right] \right\}, \tag{A.7}$$

subject to (2.13). I guess that the value function for each type of entrepreneur is:

$$\mathcal{V}_z(\omega_z) = u(\chi_z \omega_z), \quad z = i, s, \tag{A.8}$$

respectively. Where χ_z are unknown functions depending on the current and also on all other information of period t. I also guess that the policy functions for consumption and next period assets is:

$$c_{e,z} = (1-\zeta_z)\omega_z, \;\; \psi n_i' = \zeta_i \omega_i, \;\; q n_s' = \zeta_s \omega_s. \tag{A.9}$$

The first order conditions with respect to n' for entrepreneurs is given by:

$$u'(c_e) = \begin{cases} \beta \mathbb{E} \left[\pi \mathcal{V}_{\omega,i}'(\omega_{ii}') R_{ii}' + (1-\pi) \mathcal{V}_{\omega,s}'(\omega_{is}') R_{is}' \right] & \text{if} \quad z = i \\ \beta \mathbb{E} \left[\pi \mathcal{V}_{\omega,i}'(\omega_{si}') R_{si}' + (1-\pi) \mathcal{V}_{\omega,s}'(\omega_{ss}') R_{ss}' \right] & \text{if} \quad z = s \end{cases}. \tag{A.10}$$

Where $\mathcal{V}_{\omega,z}'(\cdot)$ is the derivative of the next period value function of entrepreneur with status z with respect to ω, and where returns dependent upon status have been defined in equations (2.19) in the text.

[29] Although obscured by the recursive notation employed, the value functions depend also on all prices, which - if a sequence notation would be used - could be subsumed into a subindex t in the value function.

With the guesses on policy functions in (A.9), equations (A.5) can be written as:

$$\begin{aligned} \omega'_{ii} &= R'_{ii}\zeta_i\omega_i \qquad & \omega'_{is} &= R'_{is}\zeta_i\omega_i \quad \text{if} \ \ z = i, \\ \omega'_{si} &= R'_{si}\zeta_s\omega_s \qquad & \omega'_{ss} &= R'_{ss}\zeta_s\omega_s \quad \text{if} \ \ z = s. \end{aligned} \tag{A.11}$$

Using (A.11) and the guess for the value function in (A.8), the FOC for an investor and a saver from (A.10) can be written as:

$$[(1-\zeta_i)\omega_i]^{-\sigma} = \beta\left\{\pi\left(\chi'_i R'_{ii}\right)^{1-\sigma} + (1-\pi)\left(\chi'_s R'_{is}\right)^{1-\sigma}\right\}(\zeta_i\omega_i)^{-\sigma}, \tag{A.12}$$

$$[(1-\zeta_s)\omega_s]^{-\sigma} = \beta\left\{\pi\left(\chi'_i R'_{si}\right)^{1-\sigma} + (1-\pi)\left(\chi'_s R'_{ss}\right)^{1-\sigma}\right\}(\zeta_s\omega_s)^{-\sigma}. \tag{A.13}$$

These conditions though depend on the unknowns χ_z. The Envelope Conditions come to help, they are given by:

$$\chi_z^{1-\sigma} = (1-\zeta_z)^{-\sigma}, \quad z = i, s. \tag{A.14}$$

Then the aforementioned equations can be written as equations (2.17) and (2.18) in the text.

A.3 Aggregation and Normalization

I will start first aggregating key variables for entrepreneurs, for workers aggregation is trivial. Aggregating consumption for entrepreneurs from (2.15) and (2.16):

$$C_e = (1-\zeta_i)\left\{[r + \varrho(1-\delta_k)]N\right\}\pi + (1-\zeta_s)\left\{[r + q(1-\delta_k)]N\right\}(1-\pi), \tag{A.15}$$

while aggregate investment X from (2.15) is:

$$X = \left\{ \zeta_i \frac{[r + \varrho(1-\delta_k)]N}{1-q\theta} - \frac{(1-\phi)(1-\delta_k)N}{1-\theta} \right\} \pi. \tag{A.16}$$

Workers aggregate consumption is simply $C_w = uwH$. To get a useful expression, I use market clearing in the labor market, equation (2.25) in the text.[30] Then using that equation and worker's aggregate consumption:

$$C_w = \frac{1-\alpha}{\alpha} rK. \tag{A.17}$$

For workers the other key equations are human capital accumulation and time working, the last two equations in (2.22), which are:

$$\Gamma_H = \zeta_w(1-\delta_h+\kappa), \quad u = (1-\zeta_w)\frac{1-\delta_h+\kappa}{\kappa}. \tag{A.18}$$

With $\Gamma_H = H'/H$, the growth rate of human capital.

Because the economy is growing perpetually, we need to normalize the economy to make the model stationary. To this end, I will divide variables by aggregate output, which from (2.4) is simply:

$$Y = \frac{r}{\alpha} K. \tag{A.19}$$

[30]Note also that from the definition of r in (2.5), aggregate labor demand equals to

$$L = (1-\alpha)^{\frac{1}{\alpha}} A \left(\frac{H^{\varphi}}{w} \right)^{\frac{1}{\alpha}} K.$$

Therefore labor demand is decreasing in the wage rate and increasing in productivity and capital and because K is chosen the previous period the demand of labor is a typical downward sloping curve. Also the aggregate labor supply $L^s = uH$ at any moment of time depends on the wage rate, because u depends on the wage rate through ζ, although movements of w will in general be accompanied of changes in expectations in future wages, so to portray a positive slope labor supply function is not feasible in this case.

Let me start with entrepreneur's equations. Dividing both sides of (A.15) by Y, after imposing market clearing in the equity market, (2.11a):

$$\mathcal{C}_e = (1-\zeta_i)\alpha\left[1+\frac{\varrho}{r}(1-\delta_k)\right]\pi + (1-\zeta_s)\alpha\left[1+\frac{q}{r}(1-\delta_k)\right](1-\pi), \tag{A.20}$$

where cursive letters denote the variable divided by output. For example $\mathcal{C}_e = C_e/Y$. From aggregate investment, equation (A.16), dividing by output:

$$\mathcal{X} = [\Gamma_K - (1-\delta_k)]\frac{\alpha}{r}. \tag{A.21}$$

Where $\Gamma_K = K'/K$ is the rate of growth of capital. Note that normalized workers' consumption is constant, from (A.17), equal to $\mathcal{C}_w = 1-\alpha$. Therefore goods market clearing condition (2.11c) is:

$$\mathcal{C}_e + \mathcal{C}_w + \mathcal{X} = 1. \tag{A.22}$$

Next the rate of growth of capital, from equation (A.16), is:

$$\Gamma_K = \frac{\zeta_i}{1-q\theta}\left[r+\varrho(1-\delta_k)\right]\pi - \frac{(1-\phi)(1-\delta_k)}{1-\theta}\pi + 1 - \delta_k. \tag{A.23}$$

The relevant unknowns of the system are $\mathcal{C}_e, \mathcal{X}, \zeta_i, \zeta_s, \Gamma_K, \Gamma_H, u, \zeta_w, \Gamma_w, r$ and q. And the equations that comprises the system can be listed as follows. Normalized aggregate consumption for entrepreneurs, equation(A.20) with saving rates ζ_i and ζ_s given in equations (2.17) and (2.18) respectively.[31] Investment in physical capital as a fraction of GDP, in (A.21). The rate of growth of physical capital in (A.23). The rates of growth of human capital and time devoted to work

[31] Also, returns for the different assets that appear in those equations are defined in (2.19).

in expressions in (A.18), with ζ_w defined in (2.23).[32] And market clearing in the goods market (A.22). These are 9 equations, the system is completed by taking two subsequent periods of the definition of r, equation (2.5), and from labor market clearing, equation (2.25), again, in two subsequent periods. These are equations (2.26) in the text. These form the 11 equations for the unknowns.

Several other variables of interest can also be found, from the equations derived above. For example, the gross rate of growth of the economy, is defined from (A.19) as:

$$\Gamma_Y = \frac{r'}{r}\Gamma_K. \tag{A.24}$$

The steady state

In section 2.3 we have shown how growth is unaffected by financial considerations. To derive such a result we have found the value of several variables in steady state. These were $\bar{\Gamma}_H, \bar{\Gamma}_w, \bar{\Gamma}_K$ and $\bar{\zeta}_w$ It remains to show how the rest of the variables of the system can be found in steady state. The remaining variables are: $\bar{\mathcal{C}}_e, \bar{\mathcal{X}}, \bar{r}, \bar{q}, \bar{\zeta}_i$ and $\bar{\zeta}_s$. Mechanically, under some parametrization, values for these variables can be found by solving the system composed of (A.22) with (A.20) and (A.21), (A.23) and equations in (2.17) and (2.18) all evaluated at steady state.[33] For the calibration and the values of the parameters considered, I have found that $\bar{q} > 1$ in all exercises performed.

[32]Note that the return to human capital is defined in (2.24), and that it depends on the variable Γ'_w, the rate of growth of wages.

[33]I do not present here the values found due to space limitations. Particularly, because we would get different results for different values of σ considered, and since our focus is not in the steady state per se, I differ the analysis of the implications of different values for the risk aversion to the dynamic aspects of the model.

A.4 The equity market

Demand of equity by savers is given by $n'_s - (1-\delta_k)n$ while investors demand is $n'_i - (1-\delta_k)n$, with n'_s and n'_i given in equations (2.16) and (2.15). Investors however satisfy their financial constraint in (2.7) with equality due to the assumption $q > 1$, then:

$$n'_i - (1-\delta_k)n = (1-\theta)x - \phi(1-\delta_k)n. \tag{A.25}$$

Next period aggregate equity by savers is simply

$$N'_s - (1-\delta_k)(1-\pi)N = \zeta_s \frac{r + q(1-\delta_k)}{q}(1-\pi)N - (1-\delta)(1-\pi), \tag{A.26}$$

aggregating n'_s from (2.16). Dividing by the capital stock K, and imposing market clearing in the capital market (2.11a), we obtain normalized demand of equity in equation (3.1a) in the text. Equilibrium in the equity market is:

$$N'_s - (1-\delta_k)(1-\pi)N + N'_i - (1-\delta_k)\pi N = X. \tag{A.27}$$

As aggregate demand of claims should equal investment. Then the (net) supply of claims by investors can be obtained as:

$$X - N'_i + (1-\delta_k)\pi N = \theta X + \phi(1-\delta)\pi N. \tag{A.28}$$

Investors are selling the fraction θ of claims over new units of capital created and they are reselling up to ϕ of old units of claims. These, in equilibrium must be acquired by savers.

Investor's supply of claims is written as:

$$\theta X + \phi(1-\delta_k)N\pi = \theta\left\{\zeta_i\frac{[r+\varrho(1-\delta_k)]N}{1-q\theta} - \frac{(1-\phi)(1-\delta_k)N}{1-\theta}\right\}\pi + \phi(1-\delta_k)N\pi, \quad \text{(A.29)}$$

where X was replaced from (A.16). Replacing the value of ϱ from (2.14), dividing by the stock of capital, and imposing capital market clearing (2.11a), we obtain equation (3.1b) in the text.

In solving the model in previous sections, I have used market clearing in the goods market, equation (2.11c). But equation (A.27) could have been used instead. To see this note that from (2.11c) we have $X = Y - C_e - C_w = \alpha Y + (1-\alpha)Y - C_e - C_w$, but then:

$$X = rK + (1-\alpha)Y - C_e - C_w = rK - C_e. \quad \text{(A.30)}$$

Where the first equality uses (A.19), and the second equality uses (A.17). Then $rK - C_e$ should finance investment, but this is aggregate entrepreneur's profits minus their consumption: their savings. This is a manifestation of course of Walras' law.

A.5 Linearization of the savings rate

Here I present the linearization of the savings rate. Let me focus on equation (2.17), in sequence notation it is:

$$(1-\zeta_{i,t})^{-1} = 1 + \beta^{\frac{1}{\sigma}}\left\{\left[\pi(1-\zeta_{i,t+1})^{-\sigma}R_{ii,t+1}^{1-\sigma} + (1-\pi)(1-\zeta_{s,t+1})^{-\sigma}R_{is,t+1}^{1-\sigma}\right]\right\}^{\frac{1}{\sigma}}. \quad \text{(A.31)}$$

Simple manipulations deliver:

$$\frac{(x_{i,t}-1)^{\sigma}}{\beta} = \left[\pi(1-\zeta_{i,t+1})^{-\sigma}R_{ii,t+1}^{1-\sigma} + (1-\pi)(1-\zeta_{s,t+1})^{-\sigma}R_{is,t+1}^{1-\sigma}\right], \quad \text{(A.32)}$$

where it was used (3.2). Linearization around the steady state gives, as an approximation:

$$\begin{aligned}\hat{x}_{i,t} = \frac{\beta}{\sigma}(x_i-1)^{1-\sigma}(1-\sigma)\left[\pi\left(\frac{x_i}{R_{ii}}\right)^{\sigma}\hat{R}_{ii,t+1}+(1-\pi)\left(\frac{x_s}{R_{is}}\right)^{\sigma}\mathbb{E}_t\hat{R}_{is,t+1}\right]\\ +\beta(x_i-1)^{1-\sigma}\left[\pi\left(\frac{R_{ii}}{x_i}\right)^{1-\sigma}\hat{x}_{i,t+1}+(1-\pi)\left(\frac{R_{is}}{x_s}\right)^{1-\sigma}\mathbb{E}_t\hat{x}_{s,t+1}\right].\end{aligned} \tag{A.33a}$$

Similarly, linearizing equation (2.18) for savers:

$$\begin{aligned}\hat{x}_{s,t} = \frac{\beta}{\sigma}(x_s-1)^{1-\sigma}(1-\sigma)\left[\pi\left(\frac{x_i}{R_{si}}\right)^{\sigma}\hat{R}_{si,t+1}+(1-\pi)\left(\frac{x_s}{R_{ss}}\right)^{\sigma}\mathbb{E}_t\hat{R}_{ss,t+1}\right]\\ +\beta(x_s-1)^{1-\sigma}\left[\pi\left(\frac{R_{si}}{x_i}\right)^{1-\sigma}\hat{x}_{i,t+1}+(1-\pi)\left(\frac{R_{ss}}{x_s}\right)^{1-\sigma}\hat{x}_{s,t+1}\right].\end{aligned} \tag{A.33b}$$

The system (A.33) can be written as:

$$\begin{aligned}\hat{x}_{i,t} &= \beta(1-\sigma)\left[a_i\hat{R}_{ii,t+1}+b_i\hat{R}_{is,t+1}\right]+\beta\left[c_i\hat{x}_{i,t+1}+d_i\mathbb{E}_t\hat{x}_{s,t+1}\right],\\ \hat{x}_{s,t} &= \beta(1-\sigma)\left[a_s\hat{R}_{si,t+1}+b_s\hat{R}_{ss,t+1}\right]+\beta\left[c_s\hat{x}_{i,t+1}+d_s\mathbb{E}_t\hat{x}_{s,t+1}\right],\end{aligned} \tag{A.34}$$

where:

$$\begin{aligned}a_i = (x_i-1)^{1-\sigma}\left(\frac{x_i}{R_{ii}}\right)^{\sigma}\frac{\pi}{\sigma}, b_i = (x_i-1)^{1-\sigma}\left(\frac{x_s}{R_{is}}\right)^{\sigma}\frac{1-\pi}{\sigma},\\ c_i = (x_i-1)^{1-\sigma}\left(\frac{R_{ii}}{x_i}\right)^{1-\sigma}\pi, d_i = (x_i-1)^{1-\sigma}\left(\frac{R_{is}}{x_s}\right)^{1-\sigma}(1-\pi),\\ a_s = (x_s-1)^{1-\sigma}\left(\frac{x_i}{R_{si}}\right)^{\sigma}\frac{\pi}{\sigma}, b_s = (x_s-1)^{1-\sigma}\left(\frac{x_s}{R_{ss}}\right)^{\sigma}\frac{1-\pi}{\sigma},\\ c_s = (x_s-1)^{1-\sigma}\left(\frac{R_{si}}{x_i}\right)^{1-\sigma}\pi, d_s = (x_s-1)^{1-\sigma}\left(\frac{R_{ss}}{x_s}\right)^{1-\sigma}(1-\pi).\end{aligned} \tag{A.35}$$

Note that both x_i and x_s are higher than one, therefore a_z, b_z, c_z, d_z for $z = i, s$ are all positive independent of the value of σ. In matrix form system (A.34) is written as:

$$\underbrace{\begin{bmatrix} \hat{x}_{i,t} \\ \hat{x}_{s,t} \end{bmatrix}}_{\hat{\mathbf{x}}_t} = \beta(1-\sigma) \underbrace{\begin{bmatrix} a_i & b_i & 0 & 0 \\ 0 & 0 & a_s & b_s \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \hat{R}_{ii,t+1} \\ \hat{R}_{is,t+1} \\ \hat{R}_{si,t+1} \\ \hat{R}_{ss,t+1} \end{bmatrix}}_{\hat{\mathbf{R}}_{t+1}} + \beta \underbrace{\begin{bmatrix} c_i & d_i \\ c_s & d_s \end{bmatrix}}_{\mathbf{B}} \underbrace{\begin{bmatrix} \hat{x}_{i,t+1} \\ \hat{x}_{s,t+1} \end{bmatrix}}_{\hat{\mathbf{x}}_{t+1}}. \tag{A.36}$$

Or:

$$\hat{\mathbf{x}}_t = \beta(1-\sigma)\mathbf{A}\hat{\mathbf{R}}_{t+1} + \beta\mathbf{B}\hat{\mathbf{x}}_{t+1}. \tag{A.37}$$

Iterating forward and plus the transversality condition $\lim_{s\to\infty} \beta^s \mathbf{B}^s \hat{\mathbf{x}}_{t+s} = 0$, I obtain equation (3.3) in the text.

Following similar steps, linearizing equation (2.23) for workers, I obtain:

$$\hat{x}_{w,t} = \beta(1-\sigma)a\hat{R}_{h,t+1} + \beta b \hat{x}_{w,t+1}, \tag{A.38}$$

where:

$$a = \frac{(x_w - 1)^{1-\sigma}}{\sigma} \left(\frac{x_h}{R_h}\right)^{\sigma}, \quad b = (x_w - 1)^{1-\sigma} \left(\frac{R_h}{x_w}\right)^{1-\sigma}. \tag{A.39}$$

Again because x_w is higher than one, a and b are always positive. Iterating forward (A.38) and using the transversality condition: $\lim_{s\to\infty} \beta^s b^s \hat{x}_{w,t+s} = 0$, I get (3.4) in the text.

B The model with storage good

B.1 Solving entrepreneur's problem with storage good

In this subsection, I go over some details of the solution method for entrepreneurs. Worker's problem is exactly the same as before.

Entrepreneurs maximize the same utility function (2.1) subject to constraints in (3.5)

The evolution of wealth between two periods is given by:

$$\begin{aligned} \omega'_{ii} &= [r' + \varrho'(1-\delta_k)]\, n'_i + m'_i \quad \omega'_{is} = [r' + q'(1-\delta_k)]\, n'_i + m'_i \quad \text{if} \;\; z = i, \\ \omega'_{si} &= [r' + \varrho'(1-\delta_k)]\, n'_s + m'_s \quad \omega'_{ss} = [r' + q'(1-\delta_k)]\, n'_s + m'_s \quad \text{if} \;\; z = s. \end{aligned} \tag{B.1}$$

and the Bellman equations for entrepreneurs are now:

$$\begin{aligned} \mathcal{V}_i(\omega_i) &= \max_{c_e, n', m'} \left\{ u(c_e) + \beta \left[\pi \mathcal{V}'_i(\omega'_{ii}) + (1-\pi)\mathcal{V}'_s(\omega'_{is}) \right] \right\}, \\ \mathcal{V}_s(\omega_s) &= \max_{c_e, n', m'} \left\{ u(c_e) + \beta \left[\pi \mathcal{V}'_i(\omega'_{si}) + (1-\pi)\mathcal{V}'_s(\omega'_{ss}) \right] \right\}. \end{aligned} \tag{B.2}$$

subject to (3.5). Using a similar guess than before for the value functions:

$$\mathcal{V}_z(\omega_z) = u(\chi_z \omega_z), \quad z = i, s. \tag{B.3}$$

And for policy functions:

$$c_{e,z} = (1-\zeta_z)\omega_z, \quad \psi n'_i = \zeta_i \omega_i, \quad m'_i = 0, \quad q n'_s = \mu \zeta_s \omega_s, \quad p m'_s = (1-\mu)\zeta_s \omega_s. \tag{B.4}$$

I am assuming that investors do not store any goods. This turns out to be true because producing capital is more profitable as $q > 1$ in equilibrium.

The first order conditions with respect to n' for entrepreneurs is given by:

$$u'(c_e) = \begin{cases} \beta \left[\pi \mathcal{V}'_{\omega,i}(\omega'_{ii}) R'_{ii} + (1-\pi)\mathcal{V}'_{\omega,s}(\omega'_{is}) R'_{is}\right] & \text{if} \quad z = i \\ \beta \mathbb{E}\left[\pi \mathcal{V}'_{\omega,i}(\omega'_{si}) R'_{si} + (1-\pi)\mathcal{V}'_{\omega,s}(\omega'_{ss}) R'_{ss}\right] & \text{if} \quad z = s \end{cases} . \tag{B.5}$$

While the first order condition for storage for savers:

$$u'(c_{e,s}) = \beta \left[\pi \mathcal{V}'_{\omega,i}(\omega'_{si}) R'_m + (1-\pi)\mathcal{V}'_{\omega,s}(\omega'_{ss}) R'_m\right]. \tag{B.6}$$

Where returns dependent upon status have the same expressions as in (2.19) and $R'_m = 1/p$. Note that according to the guess on policy functions for assets, equations (B.1) can be written as:

$$\begin{aligned} \omega'_{ii} &= R'_{ii}\zeta_i\omega_i \qquad & \omega'_{is} &= R'_{is}\zeta_i\omega_i \quad \text{if} \quad z = i, \\ \omega'_{si} &= [\mu R'_{si} + (1-\mu)R'_m]\zeta_s\omega_s \qquad & \omega'_{ss} &= [\mu R'_{ss} + (1-\mu)R'_m]\zeta_s\omega_s \quad \text{if} \quad z = s. \end{aligned} \tag{B.7}$$

Using (B.7) and the guess for the value function in (B.3), the FOC for an investor from (B.5) can be written as:

$$[(1-\zeta_i)\omega_i]^{-\sigma} = \beta \left\{\pi \left[\chi'_i R'_{ii}\right]^{1-\sigma} + (1-\pi)\left[\chi'_s R'_{is}\right]^{1-\sigma}\right\} (\zeta_i\omega_i)^{-\sigma}. \tag{B.8}$$

Let me work with savers' conditions. From (B.5) and (B.6) for savers, and using the guess in (B.3), I get:

$$0 = \pi \left(\chi'_i\right)^{1-\sigma} \left(\omega'_{si}\right)^{-\sigma} \left[R'_{si} - R'_m\right] + (1-\pi)\left(\chi'_s\right)^{1-\sigma} \left(\omega'_{ss}\right)^{-\sigma} \left[R'_{ss} - R'_m\right]. \tag{B.9}$$

Using (B.7) we can rewrite this condition as:

$$0 = \pi(\chi_i')^{1-\sigma}[\mu R_{si}' + (1-\mu)R_m']^{-\sigma}(R_{si}' - R_m') + (1-\pi)(\chi_s')^{1-\sigma}[\mu R_{ss}' + (1-\mu)R_m']^{-\sigma}(R_{ss}' - R_m'). \tag{B.10}$$

Next, multiplying by μ the FOC for a saver in (B.5), multiplying by $(1-\mu)$ the FOC in (B.6) and summing up, we obtain:

$$c_{e,s}^{-\sigma} = \beta\{\pi(\chi_i')^{1-\sigma}(\omega_{si}')^{-\sigma}[\mu R_{si}' + (1-\mu)R_m'] + (1-\pi)\left(\chi_s'\right)^{1-\sigma}(\omega_{ss}')^{-\sigma}[\mu R_{ss}' + (1-\mu)R_m']\}. \tag{B.11}$$

Using (B.4) and (B.7) we obtain:

$$(1-\zeta_s)^{-\sigma} = \beta\{\pi(\chi_i')^{1-\sigma}[\mu R_{si}' + (1-\mu)R_m']^{1-\sigma} + (1-\pi)(\chi_s')^{1-\sigma}[\mu R_{ss}' + (1-\mu)R_m']^{1-\sigma}\}\zeta_s^{-\sigma}. \tag{B.12}$$

Optimality conditions for assets and storage for investors and savers are given by equations (B.8), (B.10) and (B.12). These conditions though depend on the unknowns χ_z. Then again using the Envelope Conditions:

$$\chi_z^{1-\sigma} = (1-\zeta_z)^{-\sigma}, \quad z = i, s. \tag{B.13}$$

The relevant equations can be written as (2.17), (3.8) and (3.9) in the text.

B.2 The dynamic system with storage good

Having shown some of the details of the method of solution, I now present the equations that form the system of equations that comprise the model. I will omit the details as aggregation and market clearing are very similar to the case of no storage good.

The constant r for the BGP:

$$\frac{r'}{r}\Gamma_K = \Gamma'_w \frac{u'}{u}\Gamma_H. \tag{B.14a}$$

Labor market clearing

$$\frac{r'}{r} = \left(\frac{\Gamma_H^{\varphi}}{\Gamma'_w}\right)^{\frac{1}{\alpha}}. \tag{B.14b}$$

The rate of growth of physical capital:

$$\Gamma_K = \left\{\zeta_i \frac{r + \varrho(1-\delta) + \mathcal{M}}{1 - q\theta} - \frac{(1-\phi)(1-\delta_k)}{1-\theta}\right\}\pi + 1 - \delta_k. \tag{B.14c}$$

where $\mathcal{M} = M/Y$, the amount of the storage good divided by GDP. Entrepreneur's consumption:

$$\mathcal{C}_e = \alpha(1-\zeta_i)\left[1 + \frac{\varrho}{r}(1-\delta_k) + \frac{\mathcal{M}}{r}\right]\pi + \alpha(1-\zeta_s)\left[1 + \frac{q}{r}(1-\delta_k) + \frac{\mathcal{M}}{r}\right](1-\pi). \tag{B.14d}$$

Capital investment over GDP:

$$\mathcal{X}_K = \frac{\alpha}{r}\left[\Gamma_K - (1-\delta_k)\right]. \tag{B.14e}$$

Investment in storage good over GDP:

$$\mathcal{X}_M = \frac{\alpha}{r}\left[p\Gamma_K\mathcal{M}' - \mathcal{M}\right]. \tag{B.14f}$$

Goods market clearing:

$$\mathcal{C}_e + \mathcal{C}_w + \mathcal{X}_K + \mathcal{X}_M = 1. \tag{B.14g}$$

Accumulation in the storage good:

$$p\Gamma_K \mathcal{M}' = \zeta_s(1-\mu_s)\left[r + q(1-\delta_k) + \mathcal{M}\right](1-\pi). \tag{B.14h}$$

The unknowns of the system are $r, \Gamma_w, \Gamma_H, \Gamma_K, q, u, \mu, \mathcal{M}, \zeta_i, \zeta_s, \zeta_w, \mathcal{X}_K, \mathcal{X}_M, \mathcal{C}_e$. So besides the equations in (B.14) we need 6 equations more. These are given by (2.17), (3.8) and (3.9) in the text, plus (A.18) and (2.23), that correspond to workers equations, whose problem has not changed.

The model is then first solved in steady state with the same parametrization as in the previous section, and then I impose the liquidity crunch and trace out the equilibrium of the system over time as before.

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